Name: Date: Class: Team:

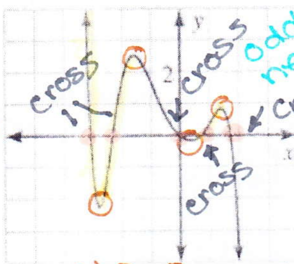
Math 3 Unit 4 Polynomial Functions Review

Directions: Read each problem carefully. Show all work and box/circle your final answer.

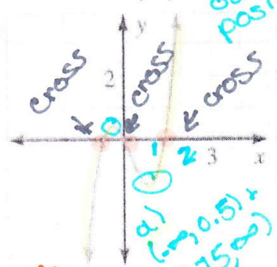
Use TIPS: Highlight/underline important information/text, Work out the problem, Write the final answer using a complete sentence. | Answer

1. Look at the graphs below and answer the following:
   a) What is the degree?
   b) How many zeros does the function have?
   c) Describe the end behavior
   d) Bonus: State the interval(s) where the function is increasing
   e) Bonus: Circle any extrema (bends/bumps)

Incr. $(-2.5, 1.5)$ & $(0.5, 1.5)$

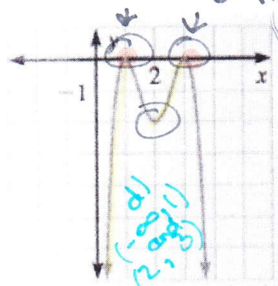


a) D=5 b) 5 roots c) $x \to -\infty, y \to \infty$ $x \to \infty, y \to -\infty$



odd positive a) D= 3 b) 3 roots c) $x \to -\infty, y \to -\infty$ $x \to \infty, y \to \infty$

d) $(-\infty, 0.5)$ & $(0.75, \infty)$



a) d = 4 b) 4 roots even negative c) $x \to -\infty, y \to -\infty$ $x \to \infty, y \to \infty$

d) $(-\infty, 1)$ and $(2, 3)$

2. Which polynomial function has zeros at 5, –4, and –3? roots $(x-5)(x+4)(x+3)$
   a. $f(x) = x^3 - 60x^2 + 2x - 23$
   b. $f(x) = x^3 + 2x^2 - 23x + 7$
   c. $f(x) = x^3 - 17x^2 - 420x + 7$
   d. $f(x) = x^3 + 2x^2 - 23x - 60$

$x-5=0$ $x+4=0$ $x+3=0$
$x=5$ $x=-4$ $x=-3$

3. Find the zeros of $f(x) = (x + 2)^6(x + 3)^4$ and state the multiplicity.
   a. –2, multiplicity 6; 4, multiplicity –3
   b. –2, multiplicity 6; –3, multiplicity 4
   c. 6, multiplicity –2; –3, multiplicity 4
   d. 6, multiplicity –2; 4, multiplicity –3

$x+2=0$ $x+3=0$
$x=-2$ multpl. of 6 $x=-3$ multpl. of 4

4. Use synthetic division to divide $-x^3 + 4x^2 - x - 3$ by $x + 2$. Write your quotient and the equation in factored form.
   a. $-x^2 + 6x - 13$
   b. $-x^2 + 2x + 11$, R –29
   c. $-x^2 + 2x + 11$
   d. $-x^2 + 6x - 13$, R 23

$x+2=0$
$x=-2$

	x^3	x^2	x	c
-2	-1	4	-1	-3
	↓	+2	-12	26
	-1	6	-13	23
	x^2	x	c	R

5. Use long division or synthetic division to divide $(x^4 + 12x^3 - 91x^2 + 26x + 20) \div (x - 5)$ Write your quotient and the equation in factored form.
   a. $x^3 + 17x^2 - 6x - 4$
   b. $x^3 - 22x^2 - 79x + 34$
   c. $x^3 + 12x^2 - 22x + 34$
   d. $x^3 - 6x^2 - 4x + 17$

$x-5=0$
$x=5$

	x^4	x^3	x^2	x	c
5	1	12	-91	26	20
	↓	5	85	-30	-20
	1	17	-6	-4	0
	x^3	x^2	x	c	R

6. Write an equation for the transformation of x^3 three units left, two units up and reflected across the x-axis.

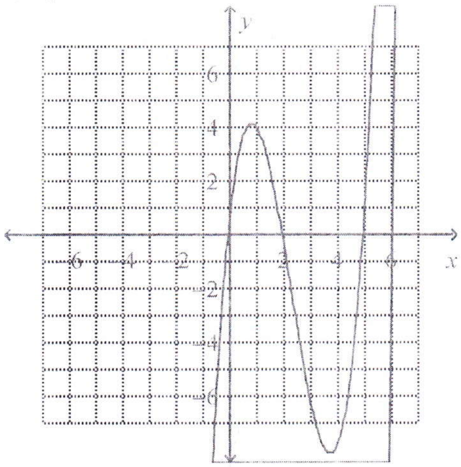
+3 +2

$y = x^3$
$y = -1(x^3)$
$y = -x^3$

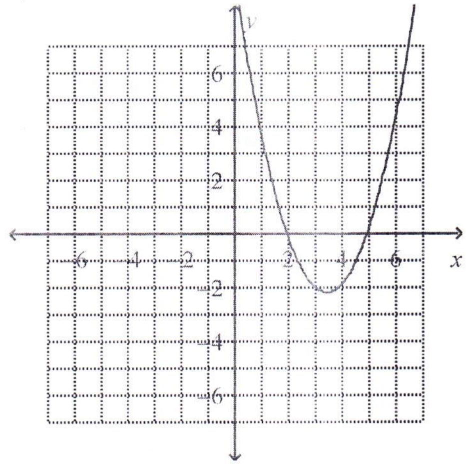
$\boxed{y = -(x+3)^3 + 2}$

. Find the zeros of $y = x(x-5)(x-2)$. Then graph the equation.

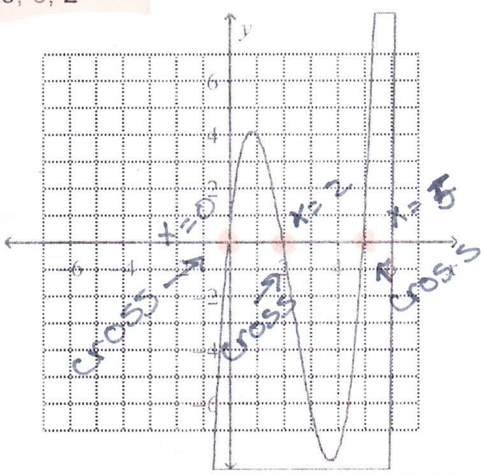
a. 5, 2, –5



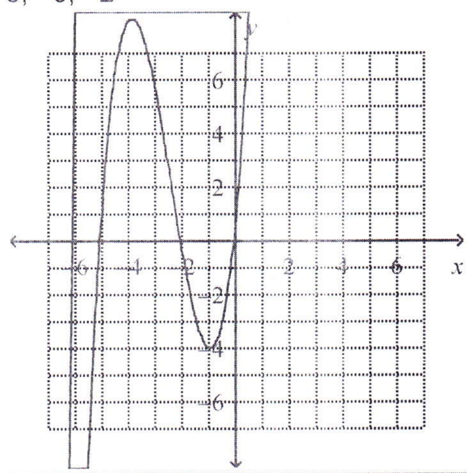
c. 5, 2



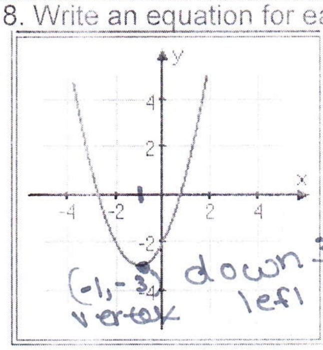
b. 0, 5, 2

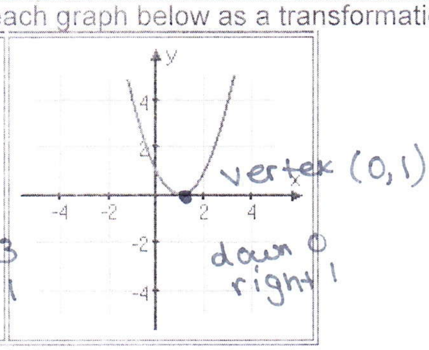


d. 0, –5, –2



8. Write an equation for each graph below as a transformation from $y = x^2$





a) $y = (x+1)^2 - 3$ b) $y = (x-1)^2$

9. Write the function of x^4 shifted 3 units down, 4 units left, a reflection over the x-axis and a horizontal compression by 3.

10. Show whether -4 is a root/zero of $g(x) = x^3 - x^2 - 14x + 24$ (Hint: substitute into equation, evaluate,and simplify to see if you get zero)

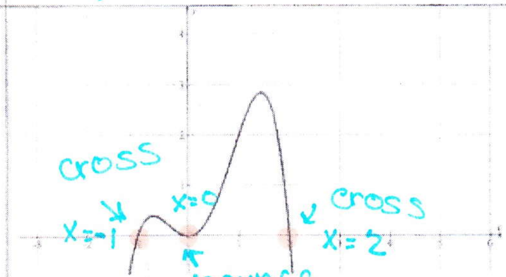
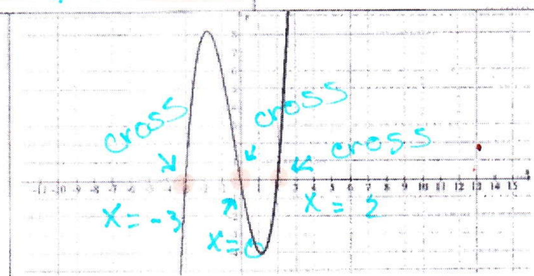
$g(-4) = (-4)^3 - (-4)^2 - 14(-4) + 24$

$-64 - 16 - (-56) + 24$

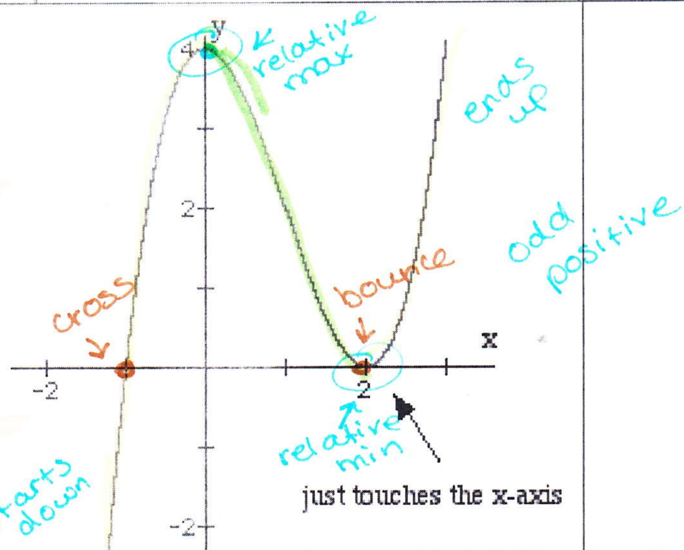
$g(-4) = 0$

yes, -4 is a zero/root.

11. Complete the following table

Convert factors to roots	$(x+5)$ $x+5=0$ $x=-5$	$(x-3)$ $x-3=0$ $x=3$	$(2x+8)$ $2x+8=0$ $x=-4$ $2x=-8$
Convert the roots to factors	X = 7 $(x-7)$ is the factor.	X = -9 $(x+9)$ is the factor.	X = 1/3 $(3x-1)$ is the factor.
Identify the FACTORS of the roots shown in the graph	 Factors: $-x^2(x+1)(x-2)$	 Factors: $x(x+3)(x-2)$	
Multiplicity of the functions graphed above	Root x = 0, multiplicity = 2 Root x = -1, multiplicity = 1 Root x = 2, multiplicity = 1	Root x = -3, multiplicity = 1 Root x = 0 multiplicity = 1 Root x = 2, multiplicity = 1	
Multiplicity of the each root in the function	$(x-3)^2(x+1)(x-2)^3$ Root: x = 3, multiplicity = 2 X = -1, multiplicity = 1 X = 2, multiplicity = 3	$(x-4)(x)(x+3)^5$ Root: x = 4, multiplicity = 1 X = 0, multiplicity = 1 X = -3, multiplicity = 5	

12. Use the graph to answer the following questions



a) Relative maximum(highest bump/bend): $(0,4)$

b) Relative minimum(lowest bump/bend): $(2,0)$

c) Increasing interval: $(-\infty,0)$ & $(2,\infty)$

d) Decreasing interval: $(0,2)$

e) Domain: $(-\infty,\infty)$

f) Range: $(-\infty,\infty)$

g) End Behavior: As $x\to-\infty$, $y\to-\infty$
As $x\to\infty$, $y\to\infty$

h) Zeros/Roots: $x=-1$
$x=2$ w/ multpl. of 2

13. Which of the following functions is a polynomial?

a. $f(x)=x$ b. $g(x)=\log_2 x$ c. $r(x)=2^x$ d. $s(x)=|x+2|$

14. Which of the following types of functions could NOT also be considered a polynomial function?

a. Linear b. Exponential c. Quadratic d. Cubic

15. Find the inverse of each function.

a. $y=\sqrt[2]{x-1}$

$x=\sqrt{y-1}$

$x^2=(\sqrt{y-1})^2$

$x^2=y-1$
$+1 \quad +1$

$x^2+1=y$

$f^{-1}(x)=x^2+1$

b. $y=x^3+7$

$x=y^3+7$

$x=y^3+7$
$-7 \quad -7$

$\sqrt[3]{x-7}=\sqrt[3]{y^3}$

$\sqrt[3]{x-7}=y$

$f^{-1}(x)=\sqrt[3]{x-7}$

16. *Matching*

Match each function on the left with its type on the right. Show the differences for each table and fill in the blank below each type.

B
1.

x	y
0	1
1	2
2	4
3	8

$\cdot 2$, $\cdot 2$, $\cdot 2$

D
2.

x	y
0	0
1	1
2	8
3	27

+1, +7, +9; +6, +2; +4

E
3.

x	y
1	0
2	1
4	2
8	3

C
4.

x	y
0	1
1	2
2	4
3	7

+1, +2, +3; +1, +1

A
5.

x	y
0	1
1	2
2	3
3	4

+1, +1, +1

a. Linear * has a constant rate of change

b. Exponential

c. Quadratic * has a linear rate of change and the 2nd difference is a constant rate of change.

d. Cubic * has a quadratic rate of change and the 3rd difference is a constant rate of change.

e. Logarithmic

17. The function $y = 187{,}900\,(1.025)^x$ represents the value of a home x years after purchase.

a. How much will the house be worth in 10 years?

Equation: $y = 187{,}900(1.025)^{10} \approx 240527.8859$

It will be worth $\approx \$240,527.89$ in 10 years.

b. When will the house be worth $300,000?

Equation:

Work: $\frac{300{,}000}{187{,}900} = \frac{187{,}900(1.025)^x}{187{,}900}$

$1.5966 = 1.025^x$

$\log_{1.025} 1.5966 = \log_{1.025} 1.025^x$

$18.95 = x$ years

It would take about 19 years to reach a value of $300,000.

18. How do you know whether a given graph is a function? How do you know if the given function is invertible?

a) It passes the vertical line test meaning there is exactly 1 output for each input.
b) I know the function is invertible b/c it will pass the horizontal line test.

19. Graph the piecewise function shown below. Assume the units are 1.

$$f(x) = \begin{cases} 5 & x \le -3 \\ -2x - 3 & x > -3 \end{cases}$$

20. The owner of a health club with 1000 members is concerned about the friendliness of his staff. He decides to survey 50 members. What type of sampling does each of the following methods represent?

B 1. Simple Random	a. Chose three workout classes and survey all members of those classes
D 2. Systematic	b. Put each name on a single slip of paper. Place all of the slips in a hat and mix well. Draw one slip out and note the name. Continue picking until the names of 50 members are selected.
A 3. Cluster	c. Ask the first 50 members who enter the club one morning.
E 4. Stratified	d. Ask every 10th person who enters the club one day.
C 5. Convenience	e. Pick the names of 25 women out of a hat; then pick the names of 25 men out of a hat.

Extra Practice:

17. Divide using long division

a) 8. $x^3 - 3x^2 + 8x - 5 \div (x-1)$

b) $4x^3 - 12x^2 - x + 15 \div (2x-3)$

Bonus Problems:

#1 The total number of video cassettes sold from 1995 to 2005 at Bob's store can be modeled by the function F(x) = $4x^3+14x^2+200x+1560$ and the number of kinds of video cassettes in Bob's store from 1995 to 2005 can be modeled by G(x) = 2x + 12, where x is the number of years since 1995. Using division, find the average number of each kind of video cassettes that Bob sold.

#2. Evaluate the polynomial $f(x) = 3x^5 - x^3 + 6x^2 - x + 1$ for $x = -2$. Explain what your answer represents.

$f(-2) = 3(-2)^5 - (-2)^3 + 6(-2)^2 - (-2) + 1$ The remainder is 61.

$f(-2) = 61$ Therefore, -2 is not a root/zero.

#3. Sketch a graph $f(x) = -4(x-1)^2(x-3)(x+8)$

X-1=0 | X-3=0 | X+8=0

X=1 | X=3 | X=-8

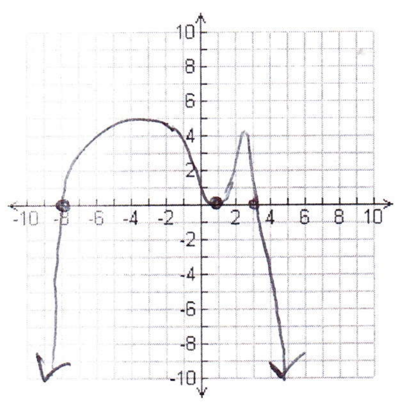
m2 bounce | cross | cross

Degree = 4 / even

L.C. = -4

starts down

ends down



Bonus #4:

The number of eggs, f(x), in a female moth is a function of her abdominal width, x, in millimeters, modeled by f(x) = $14x^3 - 17x^2 - 16x + 34$. What is the abdominal width when there are 211 eggs?

$211 = 14x^3 - 17x^2 - 16x + 34$ Input function ; observe table where y=211, $\boxed{X = 3mm}$

Bonus #5: A rectangular swimming pool is twice as long as it is wide. A small concrete walkway surrounds the pool. The walkway is a constant 2 feet wide and has an area of 196 square feet. Find the dimensions of the pool. (Hint: Write equation in factored form and use table)

$f(x) = (2x+4)(x+4) - 2x^2 = 196$ x = 15 the pool is 15ft by 30ft.

Put function in y1

Put 196 in y2

Look at table of values to find where 196 shows up in y1 and y2.

17b)

$$\begin{array}{r} 2x^2-3x-5 \\ 2x-3\overline{)4x^3-12x^2-x+15} \end{array}$$

$2x^2(2x-3) = -(4x^3-6x^2)$

$-6x^2-x$

$-3x(2x-3) = -(-6x^2+9x)$

$-10x+15$

$-5(2x-3) = -(-10x+15)$

$\varnothing$

$\boxed{2x^2-3x-5}$

$\frac{4x^3}{2x} = 2x^2$

$\frac{-6x^2}{2x} = -3x$

$\frac{-10x}{2x} = -5$

17 a)

$$\begin{array}{r} x^2-2x+6 \\ x-1\overline{)x^3-3x^2+8x-5} \end{array}$$

$\frac{x^3}{x} = x^2$

$x^2(x-1) = -(x^3-x^2)$

$-2x^2+8x$

$\frac{-2x^2}{x} = -2x$

$-2x(x-1) = -(-2x^2+2x)$

$6x-5$

$\frac{6x}{x} = 6$

$6(x-1) = -(6x-6)$

1

$\boxed{\begin{array}{c} x^2-2x+6 \text{ R. } 1 \\ x^2-2x+6+\frac{1}{x-1} \end{array}}$

Bonus #1

$$\begin{array}{r} \boxed{2x^2-5x+136} \\ 2x+12\overline{)4x^3+14x^2+200x+1560} \end{array}$$

$2x^2(2x+12) = -(4x^3+24x^2)$

$-10x^2+200x$

$-5x(2x+12) = -(-10x^2-60x)$

$260x+1560$

$130(2x+12) = -(260x+1560)$

$\varnothing$

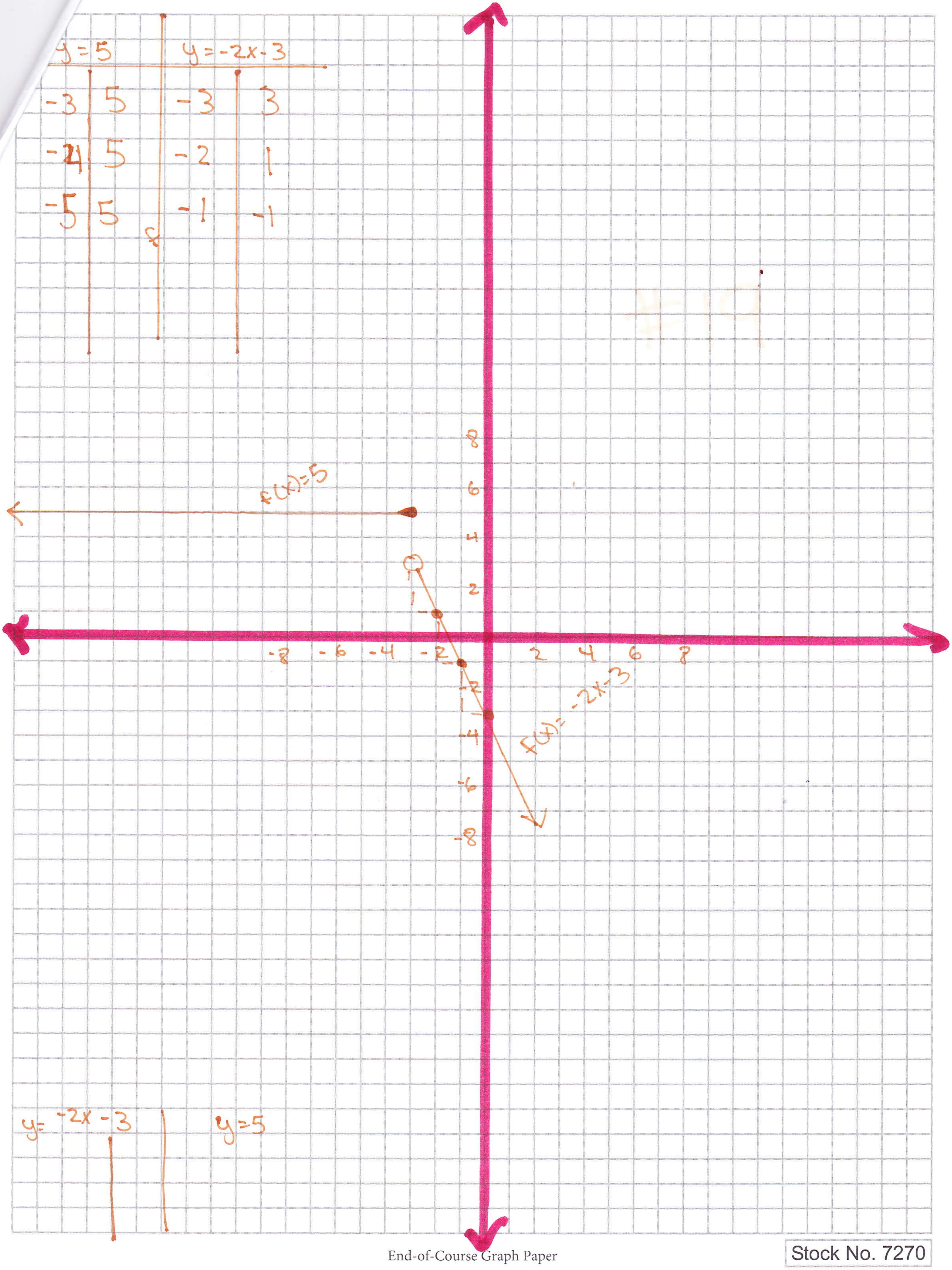
$\frac{4x^3}{2x} = 2x^2$

$\frac{-10x^2}{2x} = -5x$

$\frac{260x}{2x} = 130$

y = 5	
-3	5
-4	5
-5	5

y = -2x - 3	
-3	3
-2	1
-1	-1



y= -2x - 3

y=5