

13 Volume

What You'll Learn

- **Lessons 13-1, 13-2, and 13-3** Find volumes of prisms, cylinders, pyramids, cones, and spheres.
- **Lesson 13-4** Identify congruent and similar solids, and state the properties of similar solids.
- **Lesson 13-5** Graph solids in space, and use the Distance and Midpoint Formulas in space.

Key Vocabulary

- volume (p. 688)
- similar solids (p. 707)
- congruent solids (p. 707)
- ordered triple (p. 714)

Why It's Important

Volcanoes like those found in Lassen Volcanic National Park, California, are often shaped like cones. By applying the formula for volume of a cone, you can find the amount of material in a volcano. *You will learn more about volcanoes in Lesson 13-2.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 13.

For Lesson 13-1

Pythagorean Theorem

Find the value of the variable in each equation. (For review, see Lesson 7-2.)

1. $a^2 + 12^2 = 13^2$
2. $(4\sqrt{3})^2 + b^2 = 8^2$
3. $a^2 + a^2 = (3\sqrt{2})^2$
4. $b^2 + 3b^2 = 192$
5. $256 + 7^2 = c^2$
6. $144 + 12^2 = c^2$

For Lesson 13-2

Area of Polygons

Find the area of each regular polygon. Round to the nearest tenth. (For review, see Lesson 11-3.)

7. hexagon with side length 7.2 cm
8. hexagon with side length 7 ft
9. octagon with side length 13.4 mm
10. octagon with side length 10 in.

For Lesson 13-4

Exponential Expressions

Simplify. (For review, see page 746.)

11. $(5b)^2$
12. $\left(\frac{n}{4}\right)^2$
13. $\left(\frac{3x}{4y}\right)^2$
14. $\left(\frac{4y}{7}\right)^2$

For Lesson 13-5

Midpoint Formula

W is the midpoint of \overline{AB} . For each pair of points, find the coordinates of the third point.

(For review, see Lesson 1-3.)

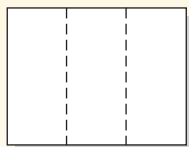
15. $A(0, -1), B(-5, 4)$
16. $A(5, 0), B(-3, 6)$
17. $A(1, -1), W(10, 10)$
18. $W(0, 0), B(-2, 2)$

FOLDABLES™ Study Organizer

Volume Make this Foldable to help you organize your notes. Begin with one sheet of $8\frac{1}{2}$ " by 11" paper.

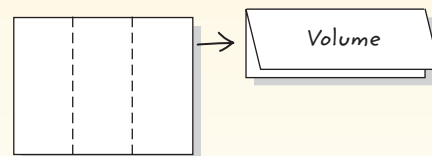
Step 1 Fold

Fold in thirds.



Step 2 Fold and Label

Fold in half lengthwise. Label as shown.



Step 3 Label

Unfold book. Draw lines along the folds and label as shown.

Prisms	Cylinders	Pyramids
Cones	Spheres	Similar

Reading and Writing As you read and study the chapter, write examples and notes about the volume of each solid and about similar solids.

Volumes of Prisms and Cylinders

What You'll Learn

- Find volumes of prisms.
- Find volumes of cylinders.

Vocabulary

- volume

Virginia SOL

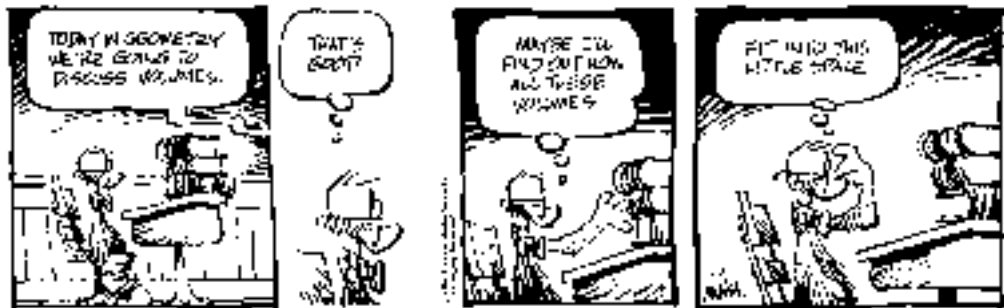
Standard G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

Standard G.14.b The student will determine how changes in one dimension of an object affect area and/or volume of the object.

How is mathematics used in comics?

Creators of comics occasionally use mathematics.

SHOE



In the comic above, the teacher is getting ready to teach a geometry lesson on volume. Shoe seems to be confused about the meaning of volume.

VOLUMES OF PRISMS The **volume** of a figure is the measure of the amount of space that a figure encloses. Volume is measured in cubic units. You can create a rectangular prism from different views of the figure to investigate its volume.



Geometry Activity

Volume of a Rectangular Prism

• Model

Use cubes to make a model of the solid with the given orthogonal drawing.



top view



left view



front view



right view

Analyze

1. How many cubes make up the prism?
2. Find the product of the length, width, and height of the prism.
3. Compare the number of cubes to the product of the length, width, and height.
4. Repeat the activity with a prism of different dimensions.
5. **Make a conjecture** about the formula for the volume of a right rectangular prism.



Study Tip

Volume and Area

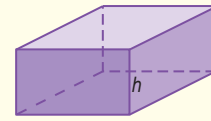
Area is two-dimensional, so it is measured in square units. Volume is three-dimensional, so it is measured in cubic units.

The Geometry activity leads to the formula for the volume of a prism.

Key Concept

Volume of a Prism

If a prism has a volume of V cubic units, a height of h units, and each base has an area of B square units, then $V = Bh$.



Area of base = B

Example 1 Volume of a Triangular Prism

Find the volume of the triangular prism.

Use the Pythagorean Theorem to find the leg of the base of the prism.

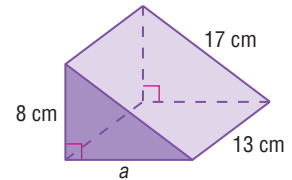
$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$

$$a^2 + 8^2 = 17^2 \quad b = 8, c = 17$$

$$a^2 + 64 = 289 \quad \text{Multiply.}$$

$$a^2 = 225 \quad \text{Subtract 64 from each side.}$$

$$a = 15 \quad \text{Take the square root of each side.}$$



Next, find the volume of the prism.

$$\begin{aligned} V &= Bh && \text{Volume of a prism} \\ &= \frac{1}{2}(8)(15)(13) && B = \frac{1}{2}(8)(15), h = 13 \\ &= 780 && \text{Simplify.} \end{aligned}$$

The volume of the prism is 780 cubic centimeters.

The volume formula can be used to solve real-world problems.

Example 2 Volume of a Rectangular Prism

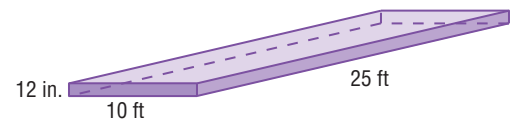
SNOW The weight of wet snow is 0.575 times the volume of snow in cubic inches divided by 144. How many pounds of wet snow would a person shovel in a rectangular driveway 25 feet by 10 feet after 12 inches of snow have fallen?

First, make a drawing.

Then convert feet to inches.

$$25 \text{ feet} = 25 \times 12 \text{ or } 300 \text{ inches}$$

$$10 \text{ feet} = 10 \times 12 \text{ or } 120 \text{ inches}$$



To find the pounds of wet snow shoveled, first find the volume of snow on the driveway.

$$\begin{aligned} V &= Bh && \text{Volume of a prism} \\ &= 300(120)(12) && B = 300(120), h = 12 \\ &= 432,000 && \text{The volume is 432,000 cubic inches.} \end{aligned}$$

Now multiply the volume by 0.575 and divide by 144.

$$\frac{0.575(432,000)}{144} = 1725 \quad \text{Simplify.}$$

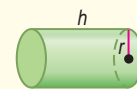
A person shoveling 12 inches of snow on a rectangular driveway 25 feet by 10 feet would shovel 1725 pounds of snow.

VOLUMES OF CYLINDERS Like the volume of a prism, the volume of a cylinder is the product of the area of the base and the height.

Key Concept

Volume of a Cylinder

If a cylinder has a volume of V cubic units, a height of h units, and the bases have radii of r units, then $V = Bh$ or $V = \pi r^2 h$.



Area of base = πr^2

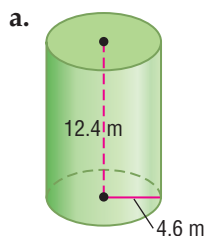
Study Tip

Square Roots

When you take the square root of each side of $h^2 = 144$, the result is actually ± 12 . Since this is a measure, the negative value is discarded.

Example 3 Volume of a Cylinder

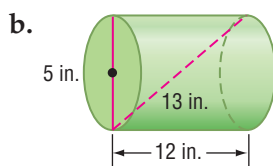
Find the volume of each cylinder.



The height h is 12.4 meters, and the radius r is 4.6 meters.

$$\begin{aligned} V &= \pi r^2 h && \text{Volume of a cylinder} \\ &= \pi(4.6^2)(12.4) && r = 4.6, h = 12.4 \\ &\approx 824.3 && \text{Use a calculator.} \end{aligned}$$

The volume is approximately 824.3 cubic meters.



The diameter of the base, the diagonal, and the lateral edge of the cylinder form a right triangle. Use the Pythagorean Theorem to find the height.

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\ h^2 + 5^2 &= 13^2 && a = h, b = 5, \text{ and } c = 13 \\ h^2 + 25 &= 169 && \text{Multiply.} \\ h^2 &= 144 && \text{Subtract 25 from each side.} \\ h &= 12 && \text{Take the square root of each side.} \end{aligned}$$

Now find the volume.

$$\begin{aligned} V &= \pi r^2 h && \text{Volume of a cylinder} \\ &= \pi(2.5^2)(12) && r = 2.5 \text{ and } h = 12 \\ &\approx 235.6 && \text{Use a calculator.} \end{aligned}$$

The volume is approximately 235.6 cubic inches.

Study Tip

Look Back

To review **oblique solids**, see Lesson 12-3.

Thus far, we have only studied the volumes of right solids. Do the formulas for volume apply to oblique solids as well as right solids?

Study the two stacks of quarters. The stack on the left represents a right cylinder, and the stack on the right represents an oblique cylinder. Since each stack has the same number of coins, with each coin the same size and shape, the two cylinders must have the same volume. Cavalieri, an Italian mathematician of the seventeenth century, was credited with making this observation first.



Study Tip

Cavalieri's Principle

This principle applies to the volumes of all solids.

Key Concept

Cavalieri's Principle

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

If a cylinder has a base with an area of B square units and a height of h units, then its volume is Bh cubic units, whether it is right or oblique.

Example 4 Volume of an Oblique Cylinder

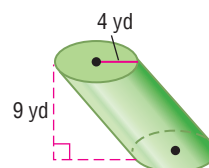
Find the volume of the oblique cylinder.

To find the volume, use the formula for a right cylinder.

$$V = \pi r^2 h \quad \text{Volume of a cylinder}$$

$$= \pi(4^2)(9) \quad r = 4, h = 9$$

$$\approx 452.4 \quad \text{Use a calculator.}$$



The volume is approximately 452.4 cubic yards.

Check for Understanding

- Concept Check**
- OPEN ENDED** List three objects that are cylinders and three that are prisms.
 - FIND THE ERROR** Che and Julia are trying to find the number of cubic feet in a cubic yard.

Che

$$V = Bh$$
$$= 3 \times 3 \times 3$$
$$= 9$$

There are 9 cubic feet in one cubic yard.

Julia

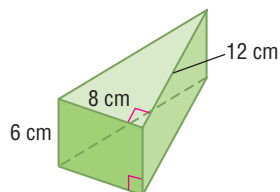
$$V = Bh$$
$$= 3 \times 3 \times 3$$
$$= 27$$

There are 27 cubic feet in one cubic yard.

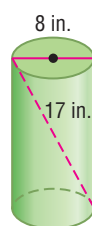
Who is correct? Explain your reasoning.

Guided Practice Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

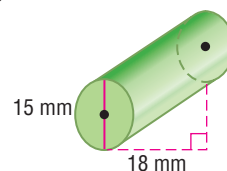
3.



4.



5.



Application

- DIGITAL CAMERA** The world's most powerful digital camera is located in New Mexico at the Apache Point Observatory. It is surrounded by a rectangular prism made of aluminum that protects the camera from wind and unwanted light. If the prism is 12 feet long, 12 feet wide, and 14 feet high, find its volume to the nearest cubic foot.

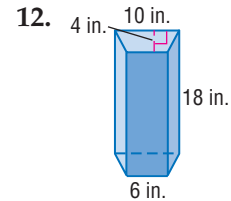
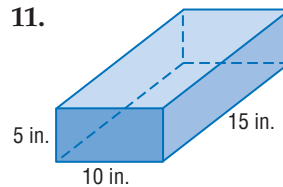
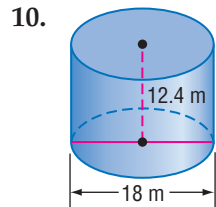
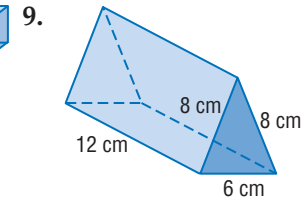
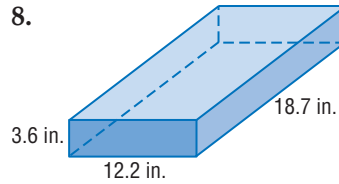
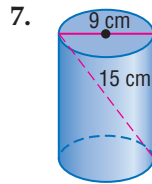
Practice and Apply

Homework Help

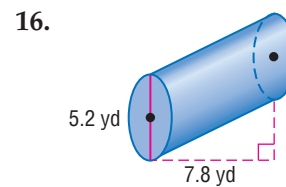
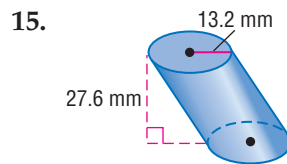
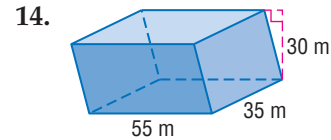
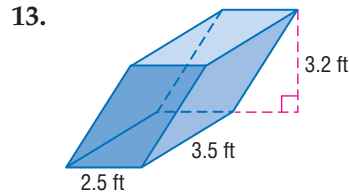
For Exercises	See Examples
7, 10, 17	3
8, 11, 18	2
9, 12	1
13–16	4

Extra Practice
See page 780.

Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.



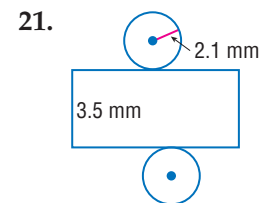
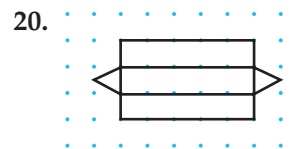
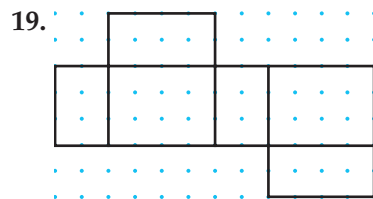
Find the volume of each oblique prism or cylinder. Round to the nearest tenth if necessary.



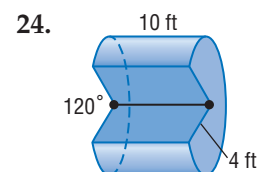
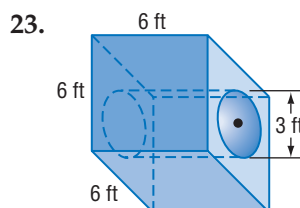
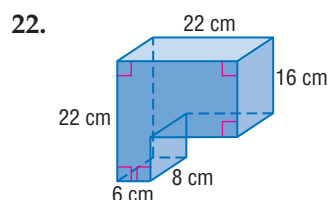
17. The volume of a cylinder is 615.8 cubic meters, and its height is 4 meters. Find the diameter of the cylinder.

18. The volume of a right rectangular prism is 1152 cubic inches, and the area of each base is 64 square inches. Find the length of the lateral edge of the prism.

Find the volume of the solid formed by each net. Round to the nearest tenth if necessary.



Find the volume of each solid. Round to the nearest tenth if necessary.



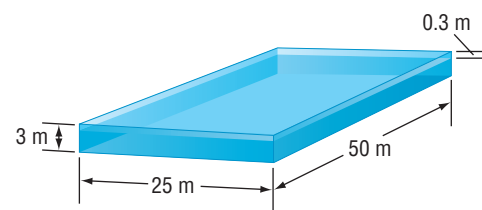
25. **MANUFACTURING** A can is 12 centimeters tall and has a diameter of 6.5 centimeters. It fits into a rubberized cylindrical holder that is 11.5 centimeters tall, including 1 centimeter, which is the thickness of the base of the holder. The thickness of the rim of the holder is 1 centimeter. What is the volume of the rubberized material that makes up the holder?

26. **ARCHITECTURE** The Marina Towers in Chicago are cylindrical shaped buildings that are 586 feet tall. There is a 35-foot-diameter cylindrical core in the center of each tower. If the core extends 40 feet above the roof of the tower, find the volume of the core.



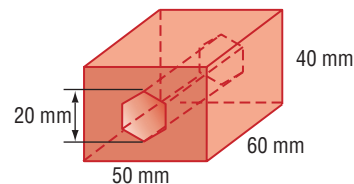
27. **AQUARIUM** The New England Aquarium in Boston, Massachusetts, has one of the world's largest cylindrical tanks. The Giant Ocean tank holds approximately 200,000 gallons and is 23 feet deep. If it takes about $7\frac{1}{2}$ gallons of water to fill a cubic foot, what is the radius of the Giant Ocean Tank?

28. **SWIMMING** A swimming pool is 50 meters long and 25 meters wide. The adjustable bottom of the pool can be up to 3 meters deep for competition and as shallow as 0.3 meter deep for recreation. The pool was filled to the recreational level, and then the floor was lowered to the competition level. If the volume of a liter of water is 0.001 cubic meter, how much water had to be added to fill the pool?



- **ENGINEERING** For Exercises 29 and 30, use the following information. Machinists make parts for intricate pieces of equipment. Suppose a part has a regular hexagonal hole drilled in a brass block.

29. Find the volume of the resulting part.
30. The *density* of a substance is its mass per unit volume. At room temperature, the density of brass is 8.0 grams per cubic centimeter. What is the mass of this block of brass?



31. **CRITICAL THINKING** Find the volume of a regular pentagonal prism with a height of 5 feet and a perimeter of 20 feet.

32. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is mathematics used in comics?

Include the following in your answer:

- the meaning of volume that Shoe has in the comic, and
- the mathematical meaning of volume.



Machinist

Some machinists use computerized equipment to cut parts. The machinist determines how fast the part should be cut based on the type of metal and harmonic vibrations.

Source: *Occupational Outlook Handbook*



Online Research

For information about a career as a machinist, visit:

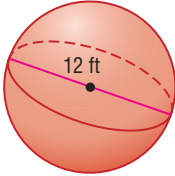
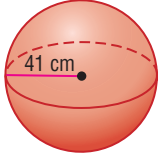
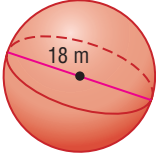
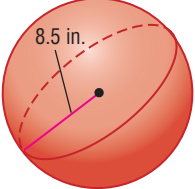
www.geometryonline.com/careers



33. A rectangular swimming pool has a volume of 16,320 cubic feet, a depth of 8 feet, and a length of 85 feet. What is the width of the swimming pool?
 (A) 24 ft (B) 48 ft (C) 192 ft (D) 2040 ft
34. **ALGEBRA** Factor $\pi r^2 h - 2\pi r h$ completely.
 (A) $\pi(r^2 h - 2rh)$ (B) $\pi r h(r - 2)$
 (C) $\pi r h(rh - 2rh)$ (D) $2\pi r(rh - h)$

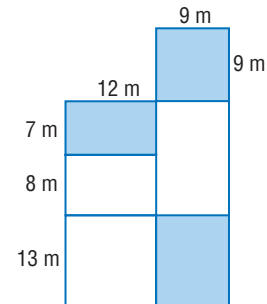
Maintain Your Skills

Mixed Review Find the surface area of each sphere. Round to the nearest tenth. (Lesson 12-7)

35.  36.  37.  38. 

Find the surface area of each cone. Round to the nearest tenth. (Lesson 12-6)

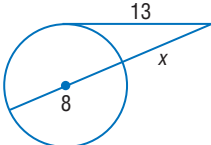
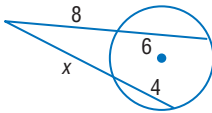
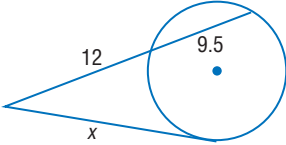
39. slant height = 11 m, radius = 6 m
 40. diameter = 16 cm, slant height = 13.5 cm
 41. radius = 5 in., height = 12 in.
 42. diameter = 14 in., height = 24 in.
43. **HOUSING** Martin lost a file at his home, and he only has time to search three of the rooms before he has to leave for work. If the shaded parts of his home will be searched, what is the probability that he finds his file? (Lesson 11-5)



Find the area of each polygon. Round to the nearest tenth. (Lesson 11-3)

44. a regular hexagon with a perimeter of 156 inches
 45. a regular octagon with an apothem 7.5 meters long and a side 6.2 meters long

Find x to the nearest tenth. Assume that any segment that appears to be tangent is tangent. (Lesson 10-7)

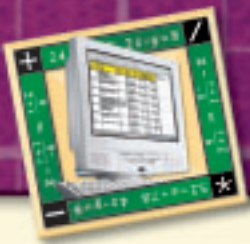
46.  47.  48. 

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the area of each polygon with given side length, s . Round to the nearest hundredth.

(To review finding areas of regular polygons, see Lesson 11-3)

49. equilateral triangle, $s = 7$ in. 50. regular hexagon, $s = 12$ cm
 51. regular pentagon, $s = 6$ m 52. regular octagon, $s = 50$ ft



Spreadsheet Investigation

A Follow-Up of Lesson 13-1

Virginia SOL Standard G.14.b The student will determine how changes in one dimension of an object affect area and/or volume of the object.

Prisms

Changing the dimensions of a prism affects the surface area and the volume of the prism. You can investigate the changes by using a spreadsheet.

- Create a spreadsheet by entering the length of the rectangular prism in column B, the width in column C, and the height in column D.
- In cell E2, enter the formula for the total surface area.
- Copy the formula in cell E2 to the other cells in column E.
- Write a formula to find the volume of the prism. Enter the formula in cell F2.
- Copy the formula in cell F2 into the other cells in column F.

Prism	l	w	h	Surface Area	Volume
1	1	2	3	22	6
2					
3					
4					
5					
6					
7					

Use your spreadsheet to find the surface areas and volumes of prisms with the dimensions given in the table below.

Prism	Length	Width	Height	Surface Area	Volume
1	1	2	3		
2	2	4	6		
3	3	6	9		
4	4	8	12		
5	8	16	24		

Exercises

1. Compare the dimensions of prisms 1 and 2, prisms 2 and 4, and prisms 4 and 5.
2. Compare the surface areas of prisms 1 and 2, prisms 2 and 4, and prisms 4 and 5.
3. Compare the volumes of prisms 1 and 2, prisms 2 and 4, and prisms 4 and 5.
4. Write a statement about the change in the surface area and volume of a prism when the dimensions are doubled.



Volumes of Pyramids and Cones

What You'll Learn

- Find volumes of pyramids.
- Find volumes of cones.



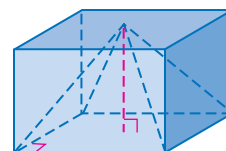
Why do architects use geometry?

The Transamerica Pyramid is the tallest skyscraper in San Francisco. The 48-story building is a square pyramid. The building was designed to allow more light to reach the street.

Virginia SOL

Standard G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

VOLUMES OF PYRAMIDS The pyramid and the prism at the right share a base and have the same height. As you can see, the volume of the pyramid is less than the volume of the prism.



Geometry Activity

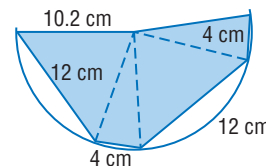
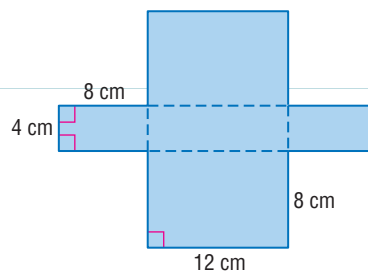
Investigating the Volume of a Pyramid

Activity

- Draw each net on card stock.
- Cut out the nets. Fold on the dashed lines.
- Tape the edges together to form models of the solids with one face removed.
- Estimate how much greater the volume of the prism is than the volume of the pyramid.
- Fill the pyramid with rice. Then pour this rice into the prism. Repeat until the prism is filled.

Analyze

1. How many pyramids of rice did it take to fill the prism?
2. Compare the areas of the bases of the prism and pyramid.
3. Compare the heights of the prism and the pyramid.
4. Make a conjecture about the formula for the volume of a pyramid.

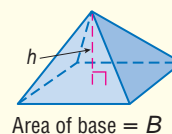


The Geometry Activity leads to the formula for the volume of a pyramid.

Key Concept

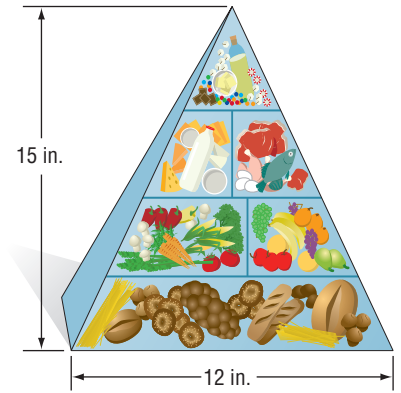
If a pyramid has a volume of V cubic units, a height of h units, and a base with an area of B square units, then $V = \frac{1}{3}Bh$.

Volume of a Pyramid



Example 1 Volume of a Pyramid

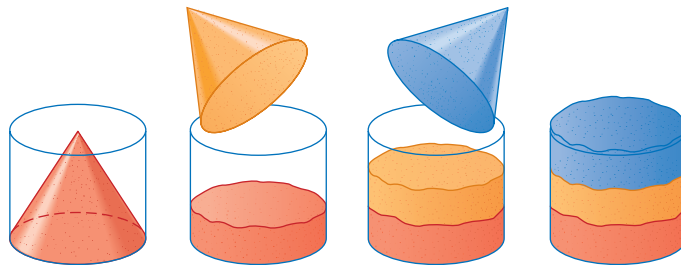
NUTRITION Travis is making a plaster model of the Food Guide Pyramid for a class presentation. The model is a square pyramid with a base edge of 12 inches and a height of 15 inches. Find the volume of plaster needed to make the model.



$$\begin{aligned} V &= \frac{1}{3}Bh && \text{Volume of a pyramid} \\ &= \frac{1}{3}s^2h && B = s^2 \\ &= \frac{1}{3}(12^2)(15) && s = 12, h = 15 \\ &= 720 && \text{Multiply.} \end{aligned}$$

Travis needs 720 cubic inches of plaster to make the model.

VOLUMES OF CONES The derivation of the formula for the volume of a cone is similar to that of a pyramid. If the areas of the bases of a cone and a cylinder are the same and if the heights are equal, then the volume of the cylinder is three times as much as the volume of the cone.



Study Tip

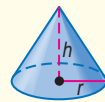
Volume Formulas

Notice how the formulas for the volumes of a cone and pyramid are the same. However, the area of the base of a cone is πr^2 .

Key Concept

If a right circular cone has a volume of V cubic units, a height of h units, and the base has a radius of r units, then $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2h$.

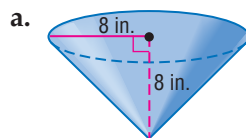
Volume of a Cone



Area of base = πr^2

Example 2 Volumes of Cones

Find the volume of each cone.

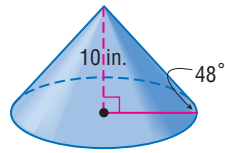


$$\begin{aligned} V &= \frac{1}{3}\pi r^2h && \text{Volume of a cone} \\ &= \frac{1}{3}\pi(8^2)(8) && r = 8, h = 8 \\ &\approx 536.165 && \text{Use a calculator.} \end{aligned}$$

The volume of the cone is approximately 536.2 cubic inches.



b.



Use trigonometry to find the radius of the base.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} \quad \text{Definition of tangent}$$

$$\tan 48^\circ = \frac{10}{r} \quad A = 48^\circ, \text{ opposite} = 10, \text{ and adjacent} = r$$

$$r = \frac{10}{\tan 48^\circ} \quad \text{Solve for } r.$$

$$r \approx 9.0 \quad \text{Use a calculator.}$$

Now find the volume.

$$V = \frac{1}{3}Bh \quad \text{Volume of a cone}$$

$$= \frac{1}{3}\pi r^2 h \quad B = \pi r^2$$

$$\approx \frac{1}{3}\pi(9^2)(10) \quad r \approx 9, h = 10$$

$$\approx 848.992 \quad \text{Use a calculator.}$$

The volume of the cone is approximately 849.0 cubic inches.

Study Tip

Common Misconceptions

The formula for the surface area of a cone only applies to right cones. However, a right cone and an oblique cone with the same radius and height have the same volume but different surface areas.

Recall that Cavalieri's Principle applies to all solids. So, the formula for the volume of an oblique cone is the same as that of a right cone.

Example 3 Volume of an Oblique Cone

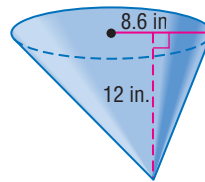
Find the volume of the oblique cone.

$$V = \frac{1}{3}Bh \quad \text{Volume of a cone}$$

$$= \frac{1}{3}\pi r^2 h \quad B = \pi r^2$$

$$= \frac{1}{3}\pi(8.6^2)(12) \quad r = 8.6, h = 12$$

$$\approx 929.4 \quad \text{Use a calculator.}$$



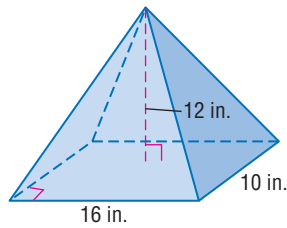
The volume of the oblique cone is approximately 929.4 cubic inches.

Check for Understanding

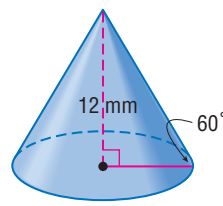
- Concept Check**
- Describe** the effect on the volumes of a cone and a pyramid if the dimensions are doubled.
 - Explain** how the volume of a pyramid is related to that of a prism with the same height and a base congruent to that of the pyramid.
 - OPEN ENDED** Draw and label two cones with different dimensions, but with the same volume.

Guided Practice Find the volume of each pyramid or cone. Round to the nearest tenth if necessary.

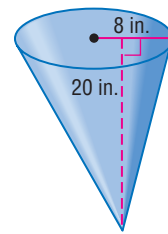
4.



5.



6.



Application 7. **MECHANICAL ENGINEERING**

The American Heritage Center at the University of Wyoming is a conical building. If the height is 77 feet, and the area of the base is about 38,000 square feet, find the volume of air that the heating and cooling systems would have to accommodate. Round to the nearest tenth.



Practice and Apply

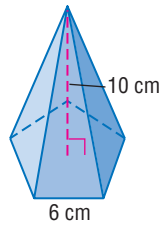
Homework Help

For Exercises	See Examples
8–10, 17–18, 24–27, 30	1
11–13, 19–23, 28–29	2
14–16	3

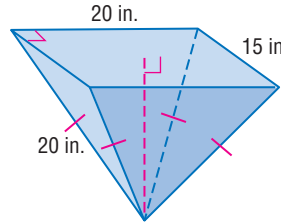
Extra Practice
See page 780.

Find the volume of each pyramid or cone. Round to the nearest tenth if necessary.

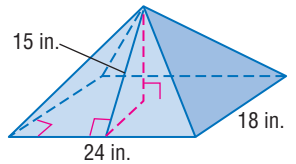
8.



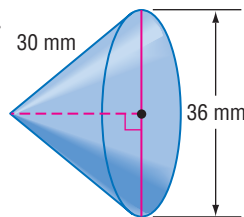
9.



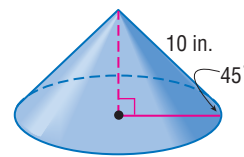
10.



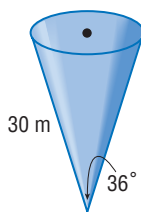
11.



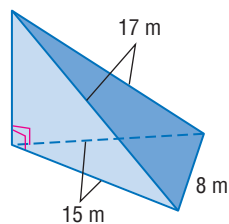
12.



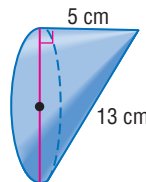
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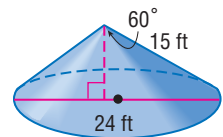
14.



15.

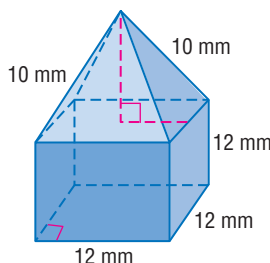


16.

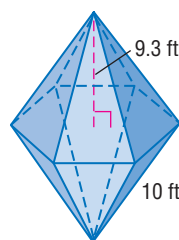


Find the volume of each solid. Round to the nearest tenth.

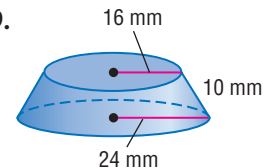
17.



18.



19.





More About . . .

Volcanoes

There are three major types of volcanoes: cinder cone, shield dome, and composite. Shield dome volcanoes are formed almost exclusively from molten lava. Composite volcanoes are formed from layers of molten lava and hardened chunks of lava.

Source: pubs.usgs.gov

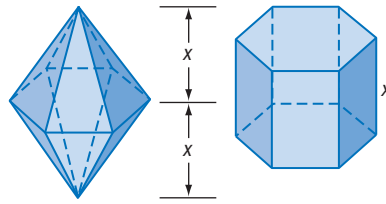
VOLCANOES For Exercises 20–23, use the following information.

The slope of a volcano is the angle made by the side of the cone and a horizontal line. Find the volume of material in each volcano, assuming that it is a solid cone.

Volcano	Location	Type	Characteristics
Mauna Loa	Hawaii, United States	shield dome	4170 m tall, 103 km across at base
Mount Fuji	Honshu, Japan	composite	3776 m tall, slope of 9°
Paricutin	Michoacán, Mexico	cinder cone	410 m tall, 33° slope
Vesuvius	Campania, Italy	composite	22.3 km across at base, 1220 m tall

20. Mauna Loa 21. Mount Fuji 22. Paricutin 23. Vesuvius

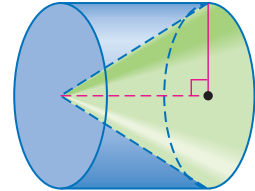
24. The shared base of the pyramids that make up the solid on the left is congruent to the base of the solid on the right. Write a ratio comparing the volumes of the solids. Explain your answer.



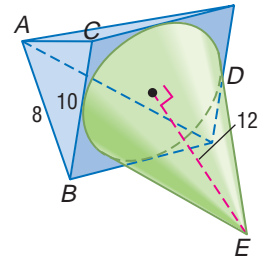
HISTORY For Exercises 25–27, use the following information.

The Great Pyramid of Khufu is a square pyramid. The lengths of the sides of the base are 755 feet. The original height was 481 feet. The current height is 449 feet.

25. Find the original volume of the pyramid.
 26. Find the present day volume of the pyramid.
 27. Compare the volumes of the pyramid. What volume of material has been lost?
 28. **PROBABILITY** What is the probability of randomly choosing a point inside the cylinder, but not inside the cone that has the same base and height as the cylinder.



29. A pyramid with a square base is next to a circular cone as shown at the right. The circular base is inscribed in the square. Isosceles $\triangle ABC$ is perpendicular to the base of the pyramid. DE is the slant height of the cone. Find the volume of the figure.



30. **ARCHITECTURE** In an attempt to rid Florida's Lower Sugarloaf Key of mosquitoes, Richter Perky built a tower to attract bats. The Perky Bat Tower is a frustum of a pyramid with a square base. Each side of the base of the tower is 15 feet long, the top is a square with sides 8 feet long, and the tower is 35 feet tall. How many cubic feet of space does the tower supply for bats? (*Hint*: Draw the pyramid that contains the frustum.)
 31. **CRITICAL THINKING** Find the volume of a regular tetrahedron with one side measuring 12 inches.

Study Tip

Look Back

To review **frustum of a pyramid**, see Lesson 12-5.

32. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do architects use geometry?

Include the following in your answer:

- compare the available office space on the first floor and on the top floor, and
- explain how a pyramidal building allows more light to reach the street than a rectangular prism building.

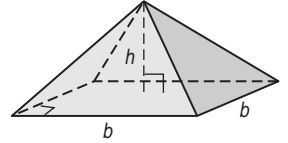
SOL/EOC Practice

Standardized Test Practice

A B C D

33. Which of the following is the volume of the square pyramid if $b = 2h$?

- (A) $\frac{h^3}{3}$ (B) $\frac{4h^3}{3}$ (C) $4h^3$ (D) $\frac{8h^3 - 4h}{3}$



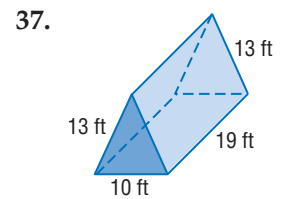
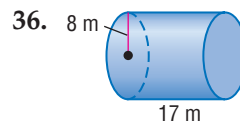
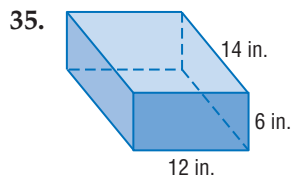
34. **ALGEBRA** If the volume of a right prism is represented by $x^3 \pm 9x$, find the factors that could represent the length, width, and height.

- (A) $x, x - 3, x + 3$ (B) $x, x - 9, x + 1$
 (C) $x, x + 9, x \pm 1$ (D) $x, x - 3, x \pm 3$

Maintain Your Skills

Mixed Review

Find the volume of each prism or cylinder. Round to the nearest tenth if necessary. (Lesson 13-1)

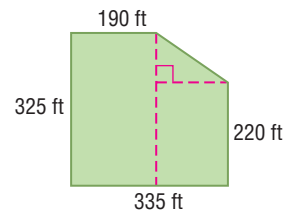


Find the surface area of each sphere. Round to the nearest tenth if necessary. (Lesson 12-7)

38. The circumference of a great circle is 86 centimeters.

39. The area of a great circle is 64.5 square yards.

40. **BASEBALL** A baseball field has the shape of a rectangle with a corner cut out as shown at the right. What is the total area of the baseball field? (Lesson 11-4)



Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate each expression. Round to the nearest hundredth.

(To review *evaluating expressions*, see page 736.)

41. $4\pi r^2$, $r = 3.4$

42. $\frac{4}{3}\pi r^3$, $r = 7$

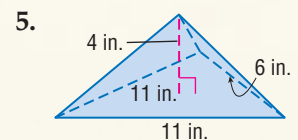
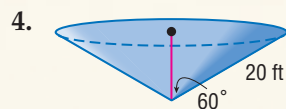
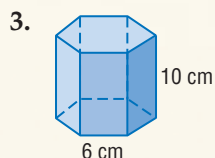
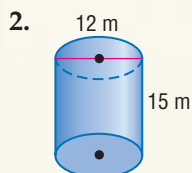
43. $4\pi r^2$, $r = 12$

Practice Quiz 1

Lessons 13-1 and 13-2

1. **FOOD** A canister of oatmeal is 10 inches tall with a diameter of 4 inches. Find the maximum volume of oatmeal that the canister can hold to the nearest tenth. (Lesson 13-1)

Find the volume of each solid. Round to the nearest tenth. (Lessons 13-1 and 13-2)



13-3

Volumes of Spheres

Virginia SOL Standard G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

What You'll Learn

- Find volumes of spheres.
- Solve problems involving volumes of spheres.

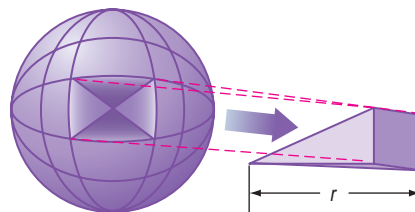
How can you find the volume of Earth?

Eratosthenes was an ancient Greek mathematician who estimated the circumference of Earth. He assumed that Earth was a sphere and estimated that the circumference was about 40,000 kilometers. From the circumference, the radius of Earth can be calculated. Then the volume of Earth can be determined.



VOLUMES OF SPHERES You can relate finding a formula for the volume of a sphere to finding the volume of a right pyramid and the surface area of a sphere.

Suppose the space inside a sphere is separated into infinitely many near-pyramids, all with vertices located at the center of the sphere. Observe that the height of these pyramids is equal to the radius r of the sphere. The sum of the areas of all the pyramid bases equals the surface area of the sphere.



Each pyramid has a volume of $\frac{1}{3}Bh$, where B is the area of its base and h is its height. The volume of the sphere is equal to the sum of the volumes of all of the small pyramids.

$$\begin{aligned}
 V &= \frac{1}{3}B_1h_1 + \frac{1}{3}B_2h_2 + \frac{1}{3}B_3h_3 + \dots + \frac{1}{3}B_nh_n && \text{Sum of the volumes of all of the pyramids} \\
 &= \frac{1}{3}B_1r + \frac{1}{3}B_2r + \frac{1}{3}B_3r + \dots + \frac{1}{3}B_nr && \text{Replace } h \text{ with } r. \\
 &= \frac{1}{3}r(B_1 + B_2 + B_3 + \dots + B_n) && \text{Distributive Property} \\
 &= \frac{1}{3}r(4\pi r^2) && \text{Replace } B_1 + B_2 + B_3 + \dots + B_n \text{ with } 4\pi r^2. \\
 &= \frac{4}{3}\pi r^3 && \text{Simplify.}
 \end{aligned}$$

Study Tip

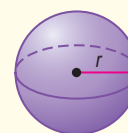
Look Back

Recall that the surface area of a sphere, $4\pi r^2$, is equal to $B_1 + B_2 + B_3 + \dots + B_n$. To review surface area of a sphere, see Lesson 12-7.

Key Concept

If a sphere has a volume of V cubic units and a radius of r units, then $V = \frac{4}{3}\pi r^3$.

Volume of a Sphere



Example 1 Volumes of Spheres

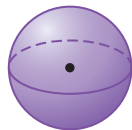
Find the volume of each sphere.

a.



$$\begin{aligned} V &= \frac{4}{3}\pi r^3 && \text{Volume of a sphere} \\ &= \frac{4}{3}\pi(24^3) && r = 24 \\ &\approx 57,905.8 \text{ in}^3 && \text{Use a calculator.} \end{aligned}$$

b. $C = 36$ cm



First find the radius of the sphere.

$$\begin{aligned} C &= 2\pi r && \text{Circumference of a circle} \\ 36 &= 2\pi r && C = 36 \\ \frac{18}{\pi} &= r && \text{Solve for } r. \end{aligned}$$

Now find the volume.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 && \text{Volume of a sphere} \\ &= \frac{4}{3}\pi\left(\frac{18}{\pi}\right)^3 && r = \frac{18}{\pi} \\ &\approx 787.9 \text{ cm}^3 && \text{Use a calculator.} \end{aligned}$$

WebQuest

The formulas for the surface area and volume of a sphere can help you find the surface area and volume of the hemisphere.

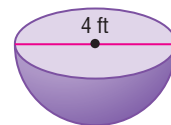
Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

Example 2 Volume of a Hemisphere

Find the volume of the hemisphere.

The volume of a hemisphere is one-half the volume of the sphere.

$$\begin{aligned} V &= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) && \text{Volume of a hemisphere} \\ &= \frac{2}{3}\pi(2^3) && r = 2 \\ &\approx 16.8 \text{ ft}^3 && \text{Use a calculator.} \end{aligned}$$



SOLVE PROBLEMS INVOLVING VOLUMES OF SPHERES Often spherical objects are contained in other solids. A comparison of the volumes is necessary to know if one object can be contained in the other.

SOL/EOC Practice

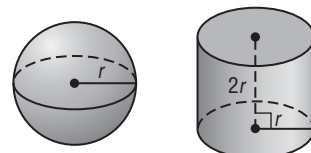
Standardized Test Practice

A B C D

Example 3 Volume Comparison

Short-Response Test Item

Compare the volumes of the sphere and the cylinder. Determine which quantity is greater.



Read the Test Item

You are asked to compare the volumes of the sphere and the cylinder.

(continued on the next page)

Solve the Test Item

Volume of the sphere: $\frac{4}{3}\pi r^3$

Volume of the cylinder: $\pi r^2 h = \pi r^2(2r)$ $h = 2r$
 $= 2\pi r^3$ **Simplify.**

Compare $2\pi r^3$ to $\frac{4}{3}\pi r^3$. Since 2 is greater than $\frac{4}{3}$, the volume of the cylinder is greater than the volume of the sphere.

Check for Understanding

Concept Check

1. Explain how to find the formula for the volume of a sphere.
2. **FIND THE ERROR** Winona and Kenji found the volume of a sphere with a radius of 12 centimeters.

Winona

$$\begin{aligned} V &= \frac{4}{3}\pi(12)^3 \\ &= 4\pi(4)^3 \\ &= 256\pi \text{ cm}^3 \end{aligned}$$

Kenji

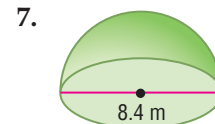
$$\begin{aligned} V &= \frac{4}{3}\pi(12)^3 \\ &= \frac{4}{3}\pi(1728) \\ &= 2304\pi \text{ cm}^3 \end{aligned}$$

Who is correct? Explain your reasoning.

Guided Practice

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

3. The radius is 13 inches long.
4. The diameter of the sphere is 12.5 centimeters.
- 5.
6. $C = 18$ cm



- 7.
8. **SHORT RESPONSE** Compare the volumes of a sphere with a radius of 5 inches and a cone with a height of 20 inches and a base with a diameter of 10 inches.

Standardized Test Practice

A B C D

Practice and Apply

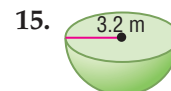
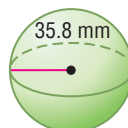
Homework Help

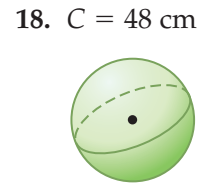
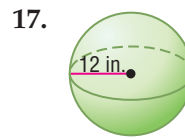
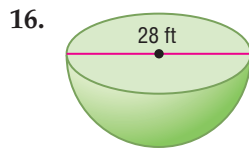
For Exercises	See Examples
9–19	1–2
20–22,	3
28–29, 32	

Extra Practice
See page 781.

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

9. The radius of the sphere is 7.62 meters.
10. The diameter of the sphere is 33 inches.
11. The diameter of the sphere is 18.4 feet.
12. The radius of the sphere is $\frac{\sqrt{3}}{2}$ centimeters.
13. $C = 24$ in.
- 14.
- 15.





19. **ASTRONOMY** The diameter of the moon is 3476 kilometers. Find the volume of the moon.

20. **SPORTS** If a golf ball has a diameter of 4.3 centimeters and a tennis ball has a diameter of 6.9 centimeters, find the difference between the volumes of the two balls.

• **FOOD** For Exercises 21 and 22, use the following information.

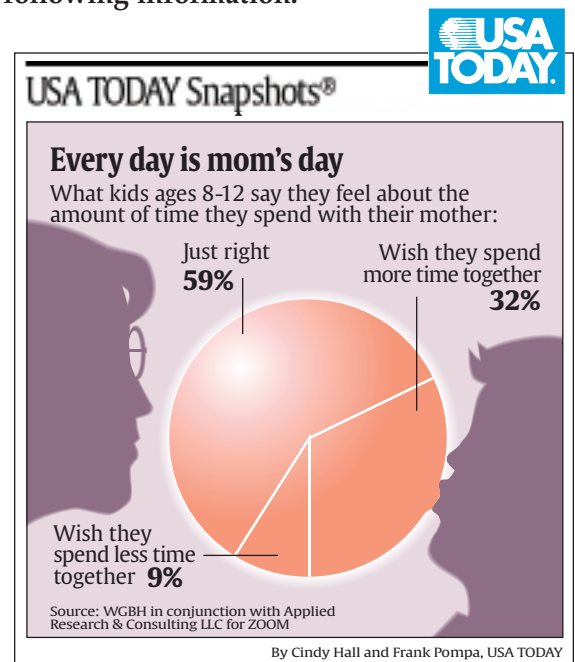
Suppose a sugar cone is 10 centimeters deep and has a diameter of 4 centimeters. A spherical scoop of ice cream with a diameter of 4 centimeters rests on the top of the cone.

21. If all the ice cream melts into the cone, will the cone overflow? Explain.
22. If the cone does not overflow, what percent of the cone will be filled?

FAMILY For Exercises 23–26, use the following information.

Suppose the bubble in the graphic is a sphere with a radius of 17 millimeters.

23. What is the volume of the bubble?
24. What is the volume of the portion of the bubble in which the kids had just the right amount of time with their mother?
25. What is the surface area of that portion of the bubble in which the kids wish they could spend more time with their mother?
26. What is the area of the two-dimensional sector of the circle in which the kids wish they could spend less time with their mother?



27. **PROBABILITY** Find the probability of choosing a point at random inside a sphere that has a radius of 6 centimeters and is inscribed in a cylinder.

28. **TENNIS** Find the volume of the empty space in a cylindrical tube of three tennis balls. The diameter of each ball is about 2.5 inches. The cylinder is 2.5 inches in diameter and is 7.5 inches tall.

29. Find the volume of a sphere that is circumscribed about a cube with a volume of 216 cubic inches.

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

30. The surface area of a sphere is 784π square inches.

31. A hemisphere has a surface area of 18.75π square meters.

More About...

Food

On average, each person in the United States consumes 16.8 pounds of ice cream per year.

Source: *Statistical Abstract of the United States*

32. **ARCHITECTURE** The Pantheon in Rome is able to contain a perfect sphere. The building is a cylinder 142 feet in diameter with a hemispherical domed roof. The total height is 142 feet. Find the volume of the interior of the Pantheon.



33. **CRITICAL THINKING** A vitamin capsule consists of a right cylinder with a hemisphere on each end. The capsule is 16 millimeters long and 4 millimeters thick. What is the volume of the capsule?
34. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you find the volume of Earth?

Include the following in your answer:

- the important dimension you must have to find the volume of Earth, and
- the radius and volume of Earth from this estimate.

35. **RESEARCH** Use the Internet or other source to find the most current calculations for the volume of Earth.

SOL/EOC Practice

Standardized
Test Practice

A B C D

36. If the radius of a sphere is increased from 3 units to 5 units, what percent would the volume of the smaller sphere be of the volume of the larger sphere?
- (A) 21.6% (B) 40% (C) 60% (D) 463%
37. **ALGEBRA** Simplify $\frac{1}{2}(4\pi r^2) + \pi r^2 h + \frac{1}{2}(4\pi r^2)$.
- (A) $\pi r^2(4 + h)$ (B) $4\pi r^2 h$ (C) $\pi r^2(9 + h)$ (D) $2\pi r^2(2 + h)$

Maintain Your Skills

Mixed Review Find the volume of each cone. Round to the nearest tenth. (Lesson 13-2)

38. height = 9.5 meters, radius = 6 meters

39. height = 7 meters, diameter = 15 meters

40. **REFRIGERATORS** A refrigerator has a volume of 25.9 cubic feet. If the interior height is 5.0 feet and the width is 2.4 feet, find the depth. (Lesson 13-1)

Write an equation for each circle. (Lesson 10-8)

41. center at $(2, -1)$, $r = 8$

42. center at $(-4, -3)$, $r = \sqrt{19}$

43. diameter with endpoints at $(5, -4)$ and $(-1, 6)$

Getting Ready for
the Next Lesson

PREREQUISITE SKILL Simplify.

(To review **simplifying expressions involving exponents**, see pages 746 and 747.)

44. $(2a)^2$

45. $(3x)^3$

46. $\left(\frac{5a}{b}\right)^2$

47. $\left(\frac{2k}{5}\right)^3$

13-4

Congruent and Similar Solids

Virginia SOL Standard G.14.a The student will use proportional reasoning to solve practical problems, given similar geometric objects; Standard G.14.b The student will determine how changes in one dimension of an object affect area and/or volume of the object.

What You'll Learn

- Identify congruent or similar solids.
- State the properties of similar solids.

Vocabulary

- similar solids
- congruent solids

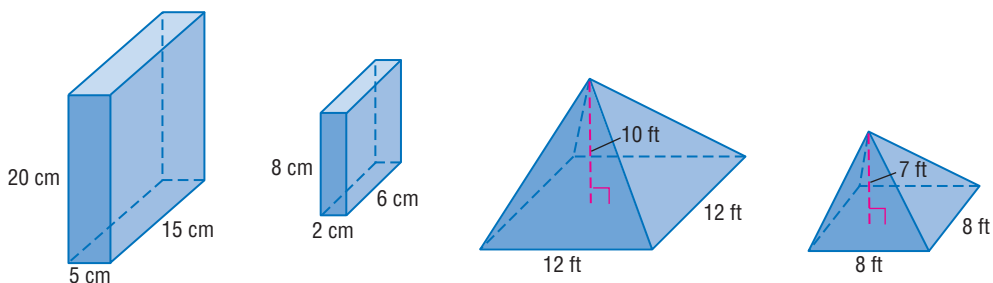
How are similar solids applied to miniature collectibles?

People collect miniatures of race cars, farm equipment, and monuments such as the Statue of Liberty. The scale factors commonly used for miniatures include 1:16, 1:24, 1:32, and 1:64. One of the smallest miniatures has a scale factor of 1:1000.

If a car is 108 inches long, then a 1:24 scale model would be $108 \div 24$ or 4.5 inches long.



CONGRUENT OR SIMILAR SOLIDS **Similar solids** are solids that have exactly the same shape but not necessarily the same size. You can determine if two solids are similar by comparing the ratios of corresponding linear measurements. In two similar polyhedra, all of the corresponding faces are similar, and all of the corresponding edges are proportional. *All spheres are similar just like all circles are similar.*



Similar Solids

Nonsimilar Solids

In the similar solids above, $\frac{8}{20} = \frac{2}{5} = \frac{6}{15}$. Recall that the ratio of the measures is called the *scale factor*.

If the ratio of corresponding measurements of two solids is 1:1, then the solids are congruent. For two solids to be congruent, all of the following conditions must be met.

Key Concept

Congruent Solids

Two solids are congruent if:

- the corresponding angles are congruent,
- the corresponding edges are congruent,
- the corresponding faces are congruent, and
- the volumes are equal.

Congruent solids are exactly the same shape and exactly the same size. They are a special case of similar solids. They have a scale factor of 1.

Study Tip

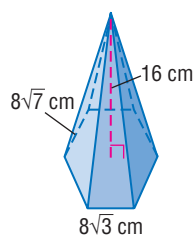
Look Back

To review **scale factor**, see Lessons 6-2 and 9-5.

Example 1 Similar and Congruent Solids

Determine whether each pair of solids are *similar*, *congruent*, or *neither*.

- a. Find the ratios between the corresponding parts of the regular hexagonal pyramids.

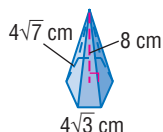


$$\frac{\text{base edge of larger pyramid}}{\text{base edge of smaller pyramid}} = \frac{8\sqrt{3}}{4\sqrt{3}} \quad \text{Substitution}$$

$$= 2 \quad \text{Simplify.}$$

$$\frac{\text{height of larger pyramid}}{\text{height of smaller pyramid}} = \frac{16}{8} \quad \text{Substitution}$$

$$= 2 \quad \text{Simplify.}$$

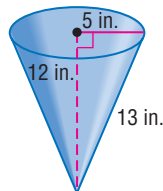
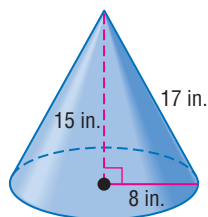


$$\frac{\text{lateral edge of larger pyramid}}{\text{lateral edge of smaller pyramid}} = \frac{8\sqrt{7}}{4\sqrt{7}} \quad \text{Substitution}$$

$$= 2 \quad \text{Simplify.}$$

The ratios of the measures are equal, so we can conclude that the pyramids are similar. Since the scale factor is not 1, the solids are not congruent.

- b. Compare the ratios between the corresponding parts of the cones.



$$\frac{\text{radius of larger cone}}{\text{radius of smaller cone}} = \frac{8}{5} \quad \text{Substitution}$$

$$\frac{\text{height of larger cone}}{\text{height of smaller cone}} = \frac{15}{12} \quad \text{Substitution}$$

Since the ratios are not the same, there is no need to find the ratio of the slant heights. The cones are not similar.

PROPERTIES OF SIMILAR SOLIDS You can investigate the relationships between similar solids using spreadsheets.



Spreadsheet Investigation

Explore Similar Solids

Collect the Data

- Step 1** In Column A, enter the labels *length*, *width*, *height*, *surface area*, *volume*, *scale factor*, *ratios of surface area*, and the *ratios of volume*. Columns B, C, D, and E will be used for four similar prisms.
- Step 2** Enter the formula for the surface area of the prism in cell B4. Copy the formula into the other cells in row 4.
- Step 3** Write a similar formula to find the volume of the prism. Copy the formula in the cells in row 5.
- Step 4** Enter the formula =C1/B1 in cell C6, enter =D1/B1 in cell D6, and so on. These formulas find the scale factor of prism B and each other solid.
- Step 5** Type the formula =C4/B4 in cell C7, type =D4/B4 in cell D7, and so on. This formula will find the ratio of the surface area of prism B to the surface areas of each of the other prisms.

Step 6 Write a formula for the ratio of the volume of prism C to the volume of prism B. Enter the formula in cell C8. Enter similar formulas in the cells in row 8.

Step 7 Use the spreadsheet to find the surface areas, volumes, and ratios for prisms with the dimensions given.

	A	B	C	D	E	F
1	length	1	2	3	4	
2	width	4	8	12	16	
3	height	6	12	18	24	
4	surface area	68	272	612	1088	
5	volume	24	192	648	1536	
6	scale factor		2	3	4	
7	ratios of surface area		4	9	16	
8	ratios of volume		8	27	64	
9						
10						

Analyze

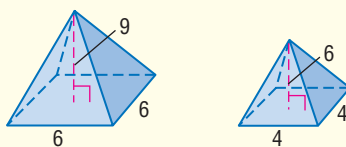
1. Compare the ratios in cells 6, 7, and 8 of columns C, D, and E. What do you observe?
2. Write a statement about the ratio of the surface areas of two solids if the scale factor is $a:b$.
3. Write a statement about the ratio of the volumes of two solids if the scale factor is $a:b$.

The Spreadsheet Investigation suggests the following theorem.

Theorem 13.1

If two solids are similar with a scale factor of $a:b$, then the surface areas have a ratio of $a^2:b^2$, and the volumes have a ratio of $a^3:b^3$.

Example:



Scale factor 3:2

Ratio of surface areas $3^2:2^2$ or 9:4

Ratio of volumes $3^3:2^3$ or 27:8

Study Tip

Standardized Tests

Problems using Theorem 13.1 are frequently on standardized tests. Be sure to note that the solids must be similar for the theorem to apply.

Example 2 Mirror Balls

ENTERTAINMENT Mirror balls are spheres that are covered with reflective tiles. One ball has a diameter of 4 inches, and another has a diameter of 20 inches.

- a. Find the scale factor of the two spheres.

Write the ratio of the corresponding measures of the spheres.

$$\frac{\text{diameter of the smaller sphere}}{\text{diameter of the larger sphere}} = \frac{4}{20} \quad \text{Substitution}$$

$$= \frac{1}{5} \quad \text{Simplify.}$$

The scale factor is 1:5.



- b. Find the ratio of the surface areas of the two spheres.

If the scale factor is $a:b$, then the ratio of the surface areas is $a^2:b^2$.

$$\begin{aligned} \frac{\text{surface area of the smaller sphere}}{\text{surface area of the larger sphere}} &= \frac{a^2}{b^2} && \text{Theorem 13.1} \\ &= \frac{1^2}{5^2} && a = 1 \text{ and } b = 5 \\ &= \frac{1}{25} && \text{Simplify.} \end{aligned}$$

The ratio of the surface areas is 1:25.

- c. Find the ratio of the volumes of the two spheres.

If the scale factor is $a:b$, then the ratio of the volumes is $a^3:b^3$.

$$\begin{aligned} \frac{\text{volume of the smaller sphere}}{\text{volume of the larger sphere}} &= \frac{a^3}{b^3} && \text{Theorem 13.1} \\ &= \frac{1^3}{5^3} && a = 1 \text{ and } b = 5 \\ &= \frac{1}{125} && \text{Simplify.} \end{aligned}$$

The ratio of the volumes of the two spheres is 1:125.

Check for Understanding

Concept Check

- OPEN ENDED** Draw and label the dimensions of a pair of cones that are similar and a pair of cones that are neither similar nor congruent.
- Explain** the relationship between the surface areas and volumes of similar solids.

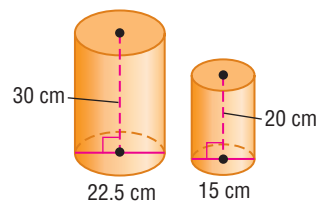
Guided Practice

Determine whether each pair of solids are *similar*, *congruent*, or *neither*.

3.

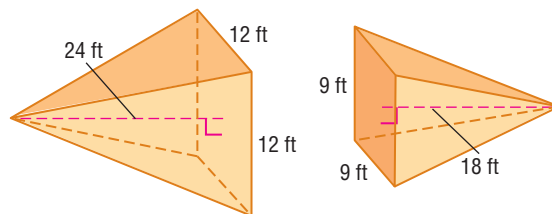


4.



For Exercises 5–7, refer to the pyramids on the right.

- Find the scale factor of the two pyramids.
- Find the ratio of the surface areas of the two pyramids.
- Find the ratio of the volumes of the two pyramids.



Application

LAWN ORNAMENTS For Exercises 8–10, use the following information.

There are two gazing balls in a garden. One has a diameter of 2 inches, and the other has a diameter of 16 inches.

- Find the scale factor of the two gazing balls.
- Determine the ratio of the surface areas of the two spheres.
- What is the ratio of the volumes of the gazing balls?

Practice and Apply

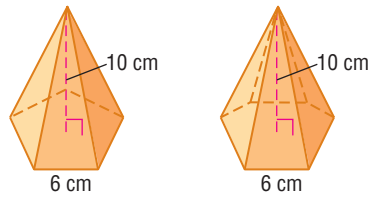
Homework Help

For Exercises	See Examples
11–16, 38, 39	1
17–37	2

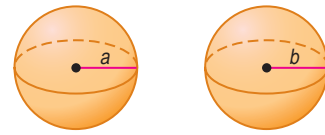
Extra Practice
See page 781.

Determine whether each pair of solids are *similar*, *congruent*, or *neither*.

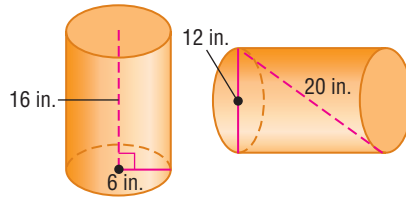
11.



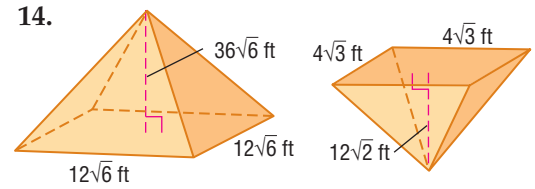
12.



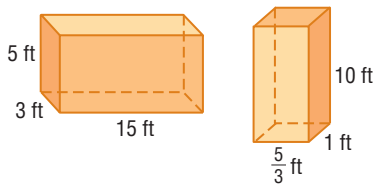
13.



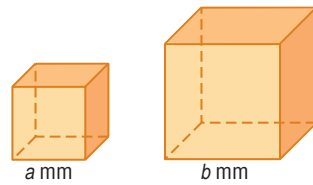
14.



15.



16.



17. **ARCHITECTURE** To encourage recycling, the people of Rome, Italy, built a model of Basilica di San Pietro from empty beverage cans. The model was built to a 1:5 scale. The model measured 26 meters high, 49 meters wide, and 93 meters long. Find the dimensions of the actual Basilica di San Pietro.

Determine whether each statement is *sometimes*, *always*, or *never* true. Justify your answer.

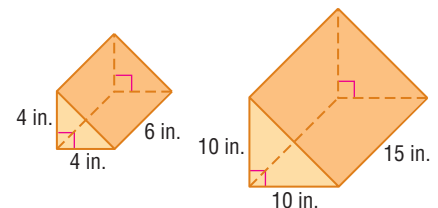
- Two spheres are similar.
- Congruent solids have equal surface areas.
- Similar solids have equal volumes.
- A pyramid is similar to a cone.
- Cones and cylinders with the same height and base are similar.
- Nonsimilar solids have different surface areas.

MINIATURES For Exercises 24–26, use the information at the left.

- If the door handle of the full-sized car is 15 centimeters long, how long is the door handle on the Micro-Car?
- If the surface area of the Micro-Car is x square centimeters, what is the surface area of the full-sized car?
- If the scale factor was 1:18 instead of 1:1000, find the length of the miniature door handle.

For Exercises 27–30, refer to the two similar right prisms.

- Find the ratio of the perimeters of the bases.
- What is the ratio of the surface areas?
- What is the ratio of the volumes?
- Suppose the volume of the smaller prism is 48 cubic inches. Find the volume of the larger prism.



More About . . .



Miniatures

The world's smallest car is a miniature version of a 1936 Model AA sedan called a DENSO Micro-Car. The scale factor is 1000:1.

Source: *The Guinness Book of Records*

31. The diameters of two similar cones are in the ratio 5 to 6. If the volume of the smaller cone is 125π cubic centimeters and the diameter of the larger cone is 12 centimeters, what is the height of the larger cone?
32. **FESTIVALS** The world's largest circular pumpkin pie was made for the Circleville Pumpkin Show in Circleville, Ohio. The diameter was 5 feet. Most pies are 8 inches in diameter. If the pies are similar, what is the ratio of the volumes?

 **Online Research Data Update** How many pies do Americans purchase in a year? Visit www.geometryonline.com/data_update to learn more.



Basketball

The National Collegiate Athletic Association (NCAA) states that the maximum circumference of a basketball for men is 30 inches. The maximum circumference of a women's basketball is 29 inches.

Source: www.ncaa.org

• **BASKETBALL** For Exercises 33–35, use the information at the left. Find the indicated ratio of the smaller ball to the larger ball.

33. scale factor 34. ratio of surface areas 35. ratio of the volumes

TOURISM For Exercises 36 and 37, use the following information.

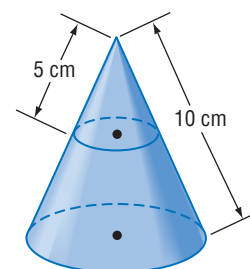
Dale Ungerer, a farmer in Hawkeye, Iowa, constructed a gigantic ear of corn to attract tourists to his farm. The ear of corn is 32 feet long and has a circumference of 12 feet. Each “kernel” is a one-gallon milk jug with a volume of 231 cubic inches.



36. If a real ear of corn is 10 inches long, what is the scale factor between the gigantic ear of corn and the similar real ear of corn?
37. Estimate the volume of a kernel of the real ear of corn.

For Exercises 38 and 39, use the following information.

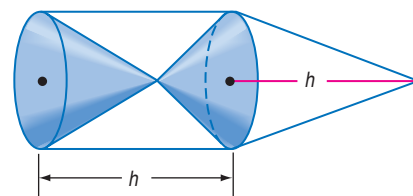
When a cone is cut by a plane parallel to its base, a cone similar to the original is formed.



38. What is the ratio of the volume of the frustum to that of the original cone? to the smaller cone?
39. What is the ratio of the lateral area of the frustum to that of the original cone? to the smaller cone?

CRITICAL THINKING For Exercises 40 and 41, refer to the figure.

40. Is it possible for the two cones inside the cylinder to be congruent? Explain.
41. Is the volume of the cone on the right equal to, greater than, or less than the sum of the volume of the cones inside the cylinder? Explain.



42. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is the geometry of similar solids applied to miniature collectibles?

Include the following in your answer:

- the scale factors that are commonly used, and
- the answer to this question: If a miniature is 4.5 inches with a scale factor of 1:24, then how long is the actual object?

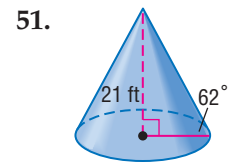
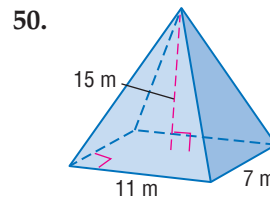
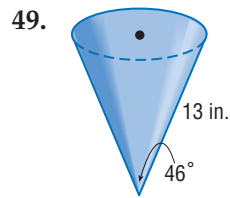
43. If the ratio of the surface areas of two similar solids is 4:9, find the ratio of the volumes.
 (A) 64:729 (B) 2:3 (C) 8:27 (D) 16:81
44. **ALGEBRA** If $xyz = 4$ and $y^2z = 5$, then what is the value of $\frac{x}{y}$?
 (A) 20.0 (B) 2.0 (C) 1.5 (D) 0.8

Maintain Your Skills

Mixed Review Find the volume of each sphere. Round to the nearest tenth. (Lesson 13-3)

45. diameter = 8 feet
 46. radius = 9.5 meters
 47. radius = 15.1 centimeters
 48. diameter = 23 inches

Find the volume of each pyramid or cone. Round to the nearest tenth. (Lesson 13-2)



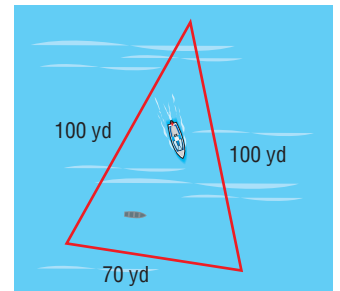
Find the radius of the base of each cylinder. Round to the nearest tenth.
 (Lesson 12-4)

52. The surface area is 430 square centimeters, and the height is 7.4 centimeters.
 53. The surface area is 224.7 square yards, and the height is 10 yards.

NAVIGATION For Exercises 54–56, use the following information.

As part of a scuba diving exercise, a 12-foot by 3-foot rectangular-shaped rowboat was sunk in a quarry. A boat takes a scuba diver to a random spot in the enclosed section of the quarry and anchors there so that the diver can search for the rowboat. (Lesson 11-5)

54. What is the approximate area of the enclosed section of the quarry?
 55. What is the area of the rowboat?
 56. What is the probability that the boat will anchor over the sunken rowboat?



Getting Ready for
the Next Lesson

PREREQUISITE SKILL Determine whether the ordered pair is on the graph of the given equation. Write *yes* or *no*. (To review **graphs in the coordinate plane**, see Lesson 1-1.)

57. $y = 3x + 5$, (4, 17) 58. $y = -4x + 1$, (-2, 9) 59. $y = 7x - 4$, (-1, 3)

Practice Quiz 2

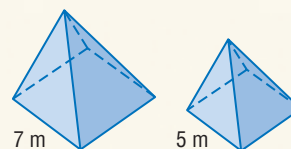
Lessons 13-3 and 13-4

Find the volume of each sphere. Round to the nearest tenth. (Lesson 13-3)

1. radius = 25.3 ft 2. diameter = 36.8 cm

The two square pyramids are similar. (Lesson 13-4)

3. Find the scale factor of the pyramids.
 4. What is the ratio of the surface areas?
 5. What is the ratio of the volumes?



13-5 Coordinates in Space

What You'll Learn

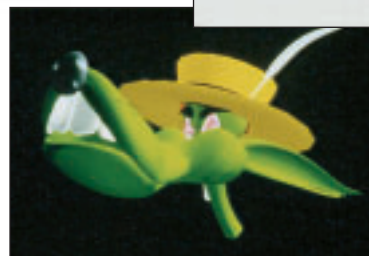
- Graph solids in space.
- Use the Distance and Midpoint Formulas for points in space.

Vocabulary

- ordered triple

How is three-dimensional graphing used in computer animation?

The initial step in computer animation is creating a three-dimensional image. A *mesh* is created first. This is an outline that shows the size and shape of the image. Then the image is *rendered*, adding color and texture. The image is animated using software. There is a way to describe the location of each point in the image.



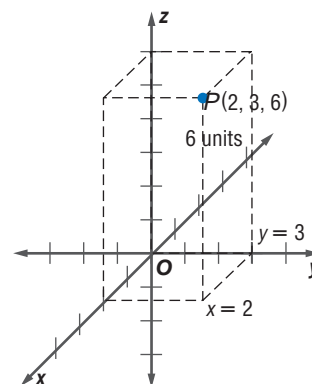
Study Tip

Reading Math

When the three planes intersect to form the three dimensional coordinate system, eight regions are formed. These regions are called *octants*.

GRAPH SOLIDS IN SPACE To describe the location of a point on the coordinate plane, we use an ordered pair of two coordinates. In space, each point requires three numbers, or coordinates, to describe its location because space has three dimensions. In space, the x -, y -, and z -axes are perpendicular to each other.

A point in space is represented by an **ordered triple** of real numbers (x, y, z) . In the figure at the right, the ordered triple $(2, 3, 6)$ locates point P . Notice that a rectangular prism is used to show perspective.



Study Tip

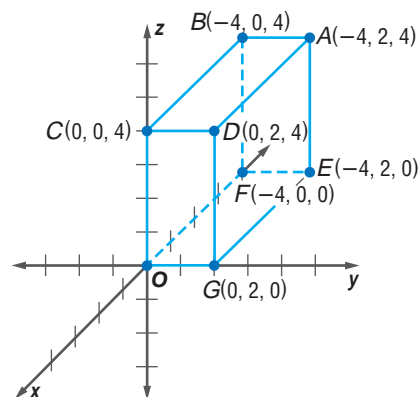
Drawing in Three Dimensions

Use the properties of a rectangular prism to correctly locate the z -coordinate. A is the vertex farthest from the origin.

Example 1 Graph a Rectangular Solid

Graph a rectangular solid that has $A(-4, 2, 4)$ and the origin as vertices. Label the coordinates of each vertex.

- Plot the x -coordinate first. Draw a segment from the origin 4 units in the negative direction.
- To plot the y -coordinate, draw a segment 2 units in the positive direction.
- Next, to plot the z -coordinate, draw a segment 4 units long in the positive direction.
- Label the coordinate A .
- Draw the rectangular prism and label each vertex.



DISTANCE AND MIDPOINT FORMULA Recall that the Distance Formula is derived from the Pythagorean Theorem. The Pythagorean Theorem can also be used to find the formula for the distance between two points in space.

Study Tip

Look Back

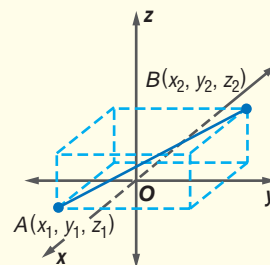
To review **Distance and Midpoint Formulas**, see Lesson 1-3.

Key Concept

Distance Formula in Space

Given two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in space, the distance between A and B is given by the following equation.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

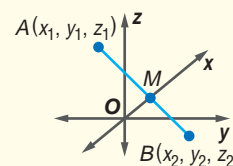


This formula is an extension of the Distance Formula in two dimensions. The Midpoint Formula can also be extended to the three-dimensions.

Key Concept

Midpoint Formula in Space

Given two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in space, the midpoint of \overline{AB} is at $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.



Example 2 Distance and Midpoint Formulas in Space

- a. Determine the distance between $T(6, 0, 0)$ and $Q(-2, 4, 2)$.

$$\begin{aligned} TQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} && \text{Distance Formula in Space} \\ &= \sqrt{[6 - (-2)]^2 + (0 - 4)^2 + (0 - 2)^2} && \text{Substitution} \\ &= \sqrt{84} \text{ or } 2\sqrt{21} && \text{Simplify.} \end{aligned}$$

- b. Determine the coordinates of the midpoint M of \overline{TQ} .

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) && \text{Midpoint Formula in Space} \\ &= \left(\frac{6 - 2}{2}, \frac{0 + 4}{2}, \frac{0 + 2}{2}\right) && \text{Substitution} \\ &= (2, 2, 1) && \text{Simplify.} \end{aligned}$$

Study Tip

Look Back

To review **translations**, see Lesson 9-2.

Example 3 Translating a Solid

ELEVATORS Suppose an elevator is 5 feet wide, 6 feet deep, and 8 feet tall. Position the elevator on the ground floor at the origin of a three dimensional space. If the distance between the floors of a warehouse is 10 feet, write the coordinates of the vertices of the elevator after going up to the third floor.

Explore Since the elevator is a rectangular prism, use positive values for x , y , and z . Write the coordinates of each corner. The points on the elevator will rise 10 feet for each floor. When the elevator ascends to the third floor, it will have traveled 20 feet.

(continued on the next page)

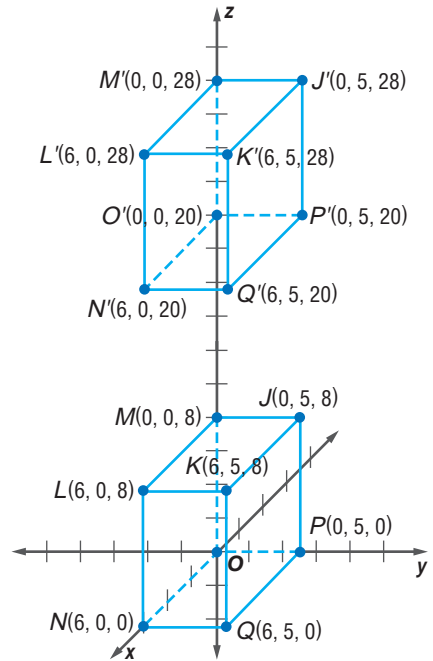


Plan

Use the translation $(x, y, z) \rightarrow (x, y, z + 20)$ to find the coordinates of each vertex of the rectangular prism that represents the elevator.

Solve

Coordinates of the vertices, (x, y, z) Preimage	Translated coordinates, $(x, y, z + 20)$ Image
$J(0, 5, 8)$	$J'(0, 5, 28)$
$K(6, 5, 8)$	$K'(6, 5, 28)$
$L(6, 0, 8)$	$L'(6, 0, 28)$
$M(0, 0, 8)$	$M'(0, 0, 28)$
$N(6, 0, 0)$	$N'(6, 0, 20)$
$O(0, 0, 0)$	$O'(0, 0, 20)$
$P(0, 5, 0)$	$P'(0, 5, 20)$
$Q(6, 5, 0)$	$Q'(6, 5, 20)$



Examine Check that the distance between corresponding vertices is 20 feet.

Matrices can be used for transformations in space such as dilations.

Study Tip

Look Back

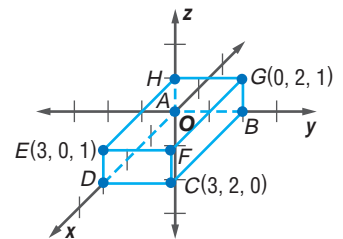
To review **matrices and transformations**, see Lesson 9-7.

Example 4 *Dilation with Matrices*

Dilate the prism by a scale factor of 2.
Graph the image under the dilation.

First, write a vertex matrix for the rectangular prism.

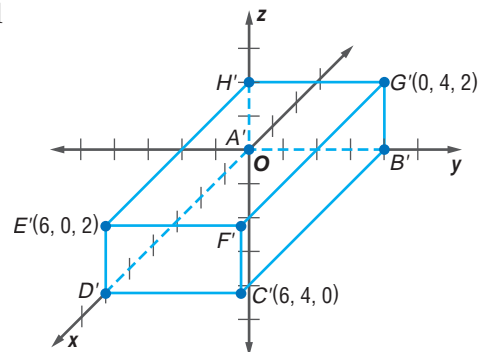
$$\begin{matrix} & A & B & C & D & E & F & G & H \\ x & \begin{bmatrix} 0 & 0 & 3 & 3 & 3 & 3 & 0 & 0 \end{bmatrix} \\ y & \begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 2 & 2 & 0 \end{bmatrix} \\ z & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{matrix}$$



Next, multiply each element of the vertex matrix by the scale factor, 2.

$$2 \begin{bmatrix} 0 & 0 & 3 & 3 & 3 & 3 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 6 & 6 & 6 & 6 & 0 & 0 \\ 0 & 4 & 4 & 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \end{bmatrix}$$

The coordinates of the vertices of the dilated image are $A'(0, 0, 0)$, $B'(0, 4, 0)$, $C'(6, 4, 0)$, $D'(6, 0, 0)$, $E'(6, 0, 2)$, $F'(6, 4, 2)$, $G'(0, 4, 2)$, and $H'(0, 0, 2)$.



Check for Understanding

Concept Check

1. Compare and contrast the number of regions on the coordinate plane and in three-dimensional coordinate space.
2. **OPEN ENDED** Draw and label the vertices of a rectangular prism that has a volume of 24 cubic units.
3. Find a counterexample for the following statement.
Every rectangular prism will be congruent to its image from any type of transformation.

Guided Practice

Graph a rectangular solid that contains the given point and the origin as vertices. Label the coordinates of each vertex.

4. $A(2, 1, 5)$
5. $P(-1, 4, 2)$

Determine the distance between each pair of points. Then determine the coordinates of the midpoint M of the segment joining the pair of points.

6. $D(0, 0, 0)$ and $E(1, 5, 7)$
7. $G(-3, -4, 6)$ and $H(5, -3, -5)$
8. The vertices of a rectangular prism are $M(0, 0, 0)$, $N(-3, 0, 0)$, $P(-3, 4, 0)$, $Q(0, 4, 0)$, $R(0, 0, 2)$, $S(0, 4, 2)$, $T(-3, 4, 2)$, and $V(-3, 0, 2)$. Dilate the prism by a scale factor of 2. Graph the image under the dilation.

Application

9. **STORAGE** A storage container is 12 feet deep, 8 feet wide, and 8 feet high. To allow the storage company to locate and identify the container, they assign ordered triples to the corners using positive x , y , and z values. If the container is stored 16 feet up and 48 feet back in the warehouse, find the ordered triples of the vertices describing the new location. Use the translation $(x, y, z) \rightarrow (x - 48, y, z + 16)$.

Practice and Apply

Homework Help

For Exercises	See Examples
10–15	1
16–21, 32–34	2
22, 25–28, 31, 35	3
23–24, 29–30	4

Extra Practice
See page 781.

Graph a rectangular solid that contains the given point and the origin as vertices. Label the coordinates of each vertex.

10. $C(-2, 2, 2)$
11. $R(3, -4, 1)$
12. $P(4, 6, -3)$
13. $G(4, 1, -3)$
14. $K(-2, -4, -4)$
15. $W(-1, -3, -6)$

Determine the distance between each pair of points. Then determine the coordinates of the midpoint M of the segment joining the pair of points.

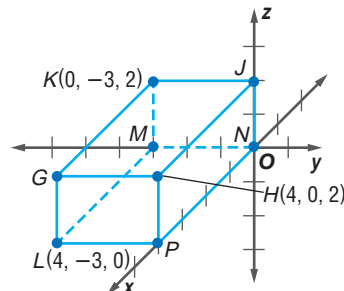
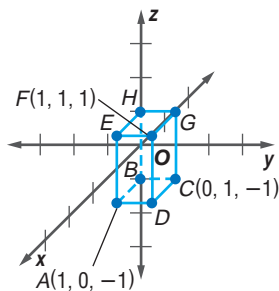
16. $K(2, 2, 0)$ and $L(-2, -2, 0)$
17. $P(-2, -5, 8)$ and $Q(3, -2, -1)$
18. $F\left(\frac{3}{5}, 0, \frac{4}{5}\right)$ and $G(0, 3, 0)$
19. $G(1, -1, 6)$ and $H\left(\frac{1}{5}, -\frac{2}{5}, 2\right)$
20. $S(6\sqrt{3}, 4, 4\sqrt{2})$ and $T(4\sqrt{3}, 5, \sqrt{2})$
21. $B(\sqrt{3}, 2, 2\sqrt{2})$ and $C(-2\sqrt{3}, 4, 4\sqrt{2})$

22. **AVIATION** An airplane at an elevation of 2 miles is 50 miles east and 100 miles north of an airport. This location can be written as $(50, 100, 2)$. A second airplane is at an elevation of 2.5 miles and is located 240 miles west and 140 miles north of the airport. This location can be written as $(-240, 140, 2.5)$. Find the distance between the airplanes to the nearest tenth of a mile.

Dilate each prism by the given scale factor. Graph the image under the dilation.

23. scale factor of 3

24. scale factor of 2



Consider a rectangular prism with the given coordinates. Find the coordinates of the vertices of the prism after the translation.

25. $P(-2, -3, 3)$, $Q(-2, 0, 3)$, $R(0, 0, 3)$, $S(0, -3, 3)$, $T(-2, 0, 0)$, $U(-2, -3, 0)$, $V(0, -3, 0)$, and $W(0, 0, 0)$; $(x, y, z) \rightarrow (x + 2, y + 5, z - 5)$

26. $A(2, 0, 1)$, $B(2, 0, 0)$, $C(2, 1, 0)$, $D(2, 1, 1)$, $E(0, 0, 1)$, $F(0, 1, 1)$, $G(0, 1, 0)$, and $H(0, 0, 0)$; $(x, y, z) \rightarrow (x - 2, y + 1, z - 1)$.

Consider a cube with coordinates $A(3, 3, 3)$, $B(3, 0, 3)$, $C(0, 0, 3)$, $D(0, 3, 3)$, $E(3, 3, 0)$, $F(3, 0, 0)$, $G(0, 0, 0)$, and $H(0, 3, 0)$. Find the coordinates of the image under each transformation. Graph the preimage and the image.

27. Use the translation $(x, y, z) \rightarrow (x + 1, y + 2, z - 2)$.

28. Use the translation $(x, y, z) \rightarrow (x - 2, y - 3, z + 2)$.

29. Dilate the cube by a factor of 2. What is the volume of the image?

30. Dilate the cube by a factor of $\frac{1}{3}$. What is the ratio of the volumes for these two cubes?

31. **RECREATION** Two hot-air balloons take off from the same site. One hot-air balloon is 12 miles west and 12 miles south of the takeoff point and 0.4 mile above the ground. The other balloon is 4 miles west and 10 miles south of the takeoff site and 0.3 mile above the ground. Find the distance between the two balloons to the nearest tenth of a mile.

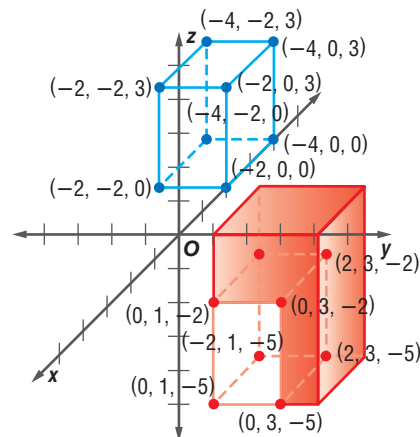
32. If $M(5, 1, 2)$ is the midpoint of segment \overline{AB} and point A has coordinates $(2, 4, 7)$, then what are the coordinates of point B ?

33. The center of a sphere is at $(4, -2, 6)$, and the endpoint of a diameter is at $(8, 10, -2)$. What are the coordinates of the other endpoint of the diameter?

34. Find the center and the radius of a sphere if the diameter has endpoints at $(-12, 10, 12)$ and $(14, -8, 2)$.

35. **GAMES** The object of a video game is to move a rectangular prism around to fit with other solids. The prism has moved to combine with the red L-shaped solid. Write the translation that moved the prism to the new location.

36. **CRITICAL THINKING** A sphere with its center at $(2, 4, 6)$ and a radius of 4 units is inscribed in a cube. Graph the cube and determine the coordinates of the vertices.



More About . . .



Recreation

Modern hot-air balloons were developed by the Montgolfier brothers in 1783. The first passengers were a sheep, a chicken, and a duck. When the animals returned safely, humans boarded the balloon.

Source: www.howstuffworks.com

37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is three-dimensional graphing used in computer animation?

Include the following in your answer:

- the purpose of using an ordered triple, and
- why three-dimensional graphing is used instead of two-dimensional graphing.

SOL/EOC Practice

Standardized Test Practice

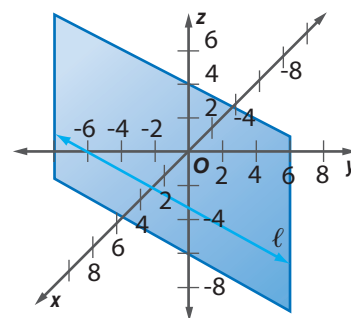
A B C D

38. The center of a sphere is at $(4, -5, 3)$, and the endpoint of a diameter is at $(5, -4, -2)$. What are the coordinates of the other endpoint of the diameter?
- (A) $(-1, -1, 5)$ (B) $(-\frac{1}{2}, -\frac{1}{2}, \frac{5}{2})$ (C) $(3, -6, 8)$ (D) $(13, -14, 4)$
39. **ALGEBRA** Solve $\sqrt{x+1} = x-1$.
- (A) 0 and 3 (B) 3 (C) -2 and 1 (D) -3 and 0

Extending the Lesson

LOCUS The locus of points in space with coordinates that satisfy the equation $y = 2x - 6$ is a plane perpendicular to the xy -plane whose intersection with the xy -plane is the graph of $y = 2x - 6$ in the xy -plane.

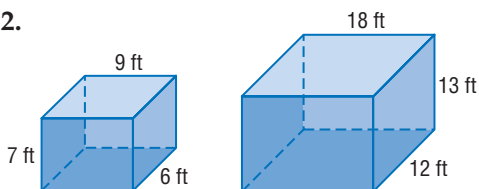
40. Describe the locus of points in space that satisfy the equation $x + y = -5$.
41. Describe the locus of points in space that satisfy the equation $x + z = 4$.



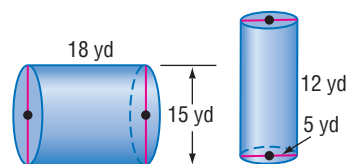
Maintain Your Skills

Mixed Review Determine whether each pair of solids are *similar*, *congruent*, or *neither*. (Lesson 13-4)

42.



43.



Find the volume of a sphere having the given radius or diameter. Round to the nearest tenth. (Lesson 13-3)

44. $r = 10$ cm

45. $d = 13$ yd

46. $r = 17.2$ m

47. $d = 29$ ft

WebQuest Internet Project

Town With Major D-Day Losses Gets Memorial

It's time to complete your project. Use the information and data you have gathered about designing your memorial. Add some additional data or pictures to your portfolio or Web page. Be sure to include your scale drawings and calculations in the presentation.



www.geometryonline.com/webquest

Vocabulary and Concept Check

congruent solids (p. 707)

ordered triple (p. 714)

similar solids (p. 707)

volume (p. 688)

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises Complete each sentence with the correct italicized term.

- You can use $V = \frac{1}{3}Bh$ to find the volume of a (*prism, pyramid*).
- (*Similar, Congruent*) solids always have the same volume.
- Every point in space can be represented by (*an ordered triple, an ordered pair*).
- $V = \pi r^2 h$ is the formula for the volume of a (*sphere, cylinder*).
- In (*similar, congruent*) solids, if $a \neq b$ and $a : b$ is the ratio of the lengths of corresponding edges, then $a^3 : b^3$ is the ratio of the volumes.
- The formula $V = Bh$ is used to find the volume of a (*prism, pyramid*).
- To find the length of an edge of a pyramid, you can use (*the Distance Formula in Space, Cavalieri's Principle*).
- You can use $V = \frac{4}{3}\pi r^3$ to find the volume of a (*cylinder, sphere*).
- To find the volume of an oblique pyramid, you can use (*Cavalieri's Principle, the Distance Formula in Space*).
- The formula $V = \frac{1}{3}Bh$ is used to find the volume of a (*cylinder, cone*).

Lesson-by-Lesson Review

13-1 Volumes of Prisms and Cylinders

See pages
688–694.

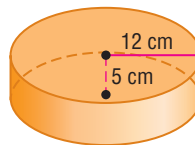
Concept Summary

- The volumes of prisms and cylinders are given by the formula $V = Bh$.

Example

Find the volume of the cylinder.

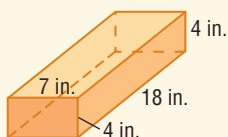
$$\begin{aligned} V &= \pi r^2 h && \text{Volume of a cylinder} \\ &= \pi(12^2)(5) && r = 12 \text{ and } h = 5 \\ &\approx 2261.9 && \text{Use a calculator.} \end{aligned}$$



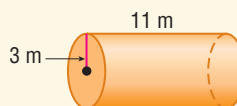
The volume is approximately 2261.9 cubic centimeters.

Exercises Find the volume of each prism or cylinder. Round to the nearest tenth if necessary. See Examples 1 and 3 on pages 689 and 690.

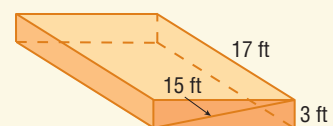
11.



12.



13.



13-2 Volumes of Pyramids and Cones

See pages 696–701.

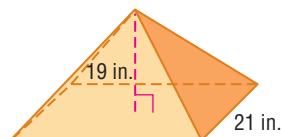
Concept Summary

- The volume of a pyramid is given by the formula $V = \frac{1}{3}Bh$.
- The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2h$.

Example

Find the volume of the square pyramid.

$$\begin{aligned} V &= \frac{1}{3}Bh && \text{Volume of a pyramid} \\ &= \frac{1}{3}(21^2)(19) && B = 21^2 \text{ and } h = 19 \\ &= 2793 && \text{Simplify.} \end{aligned}$$

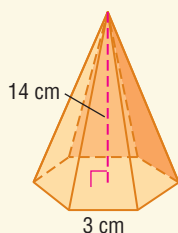


The volume of the pyramid is 2793 cubic inches.

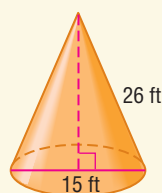
Exercises Find the volume of each pyramid or cone. Round to the nearest tenth.

See Examples 1 and 2 on pages 697 and 698.

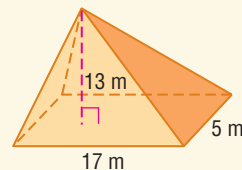
14.



15.



16.



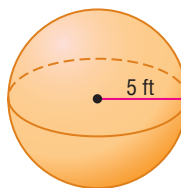
13-3

See pages 702–706.

Example

Find the volume of the sphere.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 && \text{Volume of a sphere} \\ &= \frac{4}{3}\pi(5^3) && r = 5 \\ &\approx 523.6 && \text{Use a calculator.} \end{aligned}$$



The volume of the sphere is about 523.6 cubic feet.

Exercises Find the volume of each sphere. Round to the nearest tenth.

See Example 1 on page 703.

17. The radius of the sphere is 2 feet.

18. The diameter of the sphere is 4 feet.

19. The circumference of the sphere is 65 millimeters.

20. The surface area of the sphere is 126 square centimeters.

21. The area of a great circle of the sphere is 25π square units.

13-4 Congruent and Similar Solids

See pages
707–713.

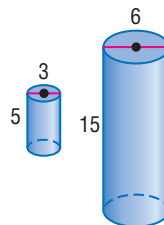
Concept Summary

- Similar solids have the same shape, but not necessarily the same size.
- Congruent solids are similar solids with a scale factor of 1.

Example

Determine whether the two cylinders are *congruent*, *similar*, or *neither*.

$$\begin{aligned} \frac{\text{diameter of larger cylinder}}{\text{diameter of smaller cylinder}} &= \frac{6}{3} && \text{Substitution} \\ &= 2 && \text{Simplify.} \\ \frac{\text{height of larger cylinder}}{\text{height of smaller cylinder}} &= \frac{15}{5} && \text{Substitution} \\ &= 3 && \text{Simplify.} \end{aligned}$$

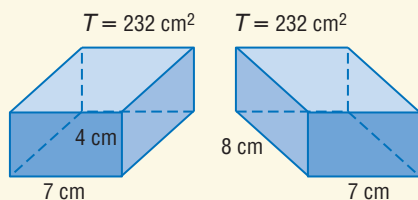


The ratios of the measures are not equal, so the cylinders are not similar.

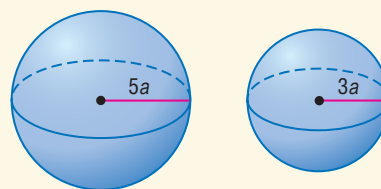
Exercises Determine whether the two solids are *congruent*, *similar*, or *neither*.

See Example 1 on page 708.

22.



23.



13-5 Coordinates in Space

See pages
714–719.

Concept Summary

- The Distance Formula in Space is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
- Given $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the midpoint of \overline{AB} is at $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

Example

Consider $\triangle ABC$ with vertices $A(13, 7, 10)$, $B(17, 18, 6)$, and $C(15, 10, 10)$. Find the length of the median from A to \overline{BC} of ABC .

$$\begin{aligned} M &= \left(\frac{17 + 15}{2}, \frac{18 + 10}{2}, \frac{6 + 10}{2}\right) && \text{Formula for the midpoint of } \overline{BC} \\ &= (16, 14, 8) && \text{Simplify.} \end{aligned}$$

\overline{AM} is the desired median, so AM is the length of the median.

$$AM = \sqrt{(16 - 13)^2 + (14 - 7)^2 + (8 - 10)^2} \text{ or } \sqrt{62} \quad \text{Distance Formula in Space}$$

Exercises Determine the distance between each pair of points. Then determine the coordinates of the midpoint M of the segment joining the pair of points.

See Example 2 on page 715.

24. $A(-5, -8, -2)$ and $B(3, -8, 4)$

25. $C(-9, 2, 4)$ and $D(-9, 9, 7)$

26. $E(-4, 5, 5)$ and the origin

27. $F(5\sqrt{2}, 3\sqrt{7}, 6)$ and $G(-2\sqrt{2}, 3\sqrt{7}, -12)$

Vocabulary and Concepts

Write the letter of the formula used to find the volume of each of the following figures.

- right cylinder
- right pyramid
- sphere

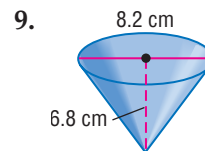
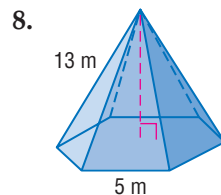
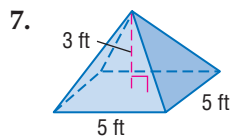
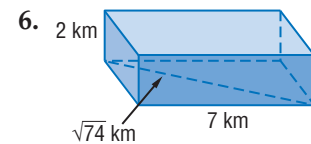
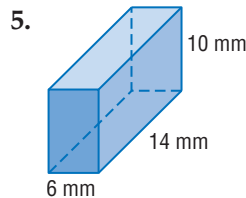
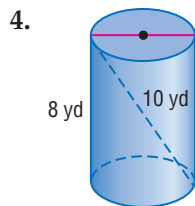
$$\text{a. } V = \frac{4}{3}\pi r^3$$

$$\text{b. } V = \pi r^2 h$$

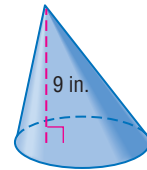
$$\text{c. } V = \frac{1}{3}Bh$$

Skills and Applications

Find the volume of each solid. Round to the nearest tenth if necessary.



10. $C = 22\pi$



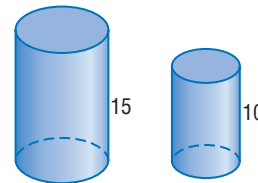
11. **SPORTS** The diving pool at the Georgia Tech Aquatic Center was used for the springboard and platform diving competitions of the 1996 Olympic Games. The pool is 78 feet long and 17 feet deep, and it is 110.3 feet from one corner on the surface of the pool to the opposite corner on the surface. If it takes about 7.5 gallons of water to fill one cubic foot of space, approximately how many gallons of water are needed to fill the diving pool?

Find the volume of each sphere. Round to the nearest tenth.

- The radius has a length of 3 cm.
- The circumference of the sphere is 34 ft.
- The surface area of the sphere is 184 in^2 .
- The area of a great circle is 157 mm^2 .

The two cylinders at the right are similar.

- Find the ratio of the radii of the bases of the cylinders.
- What is the ratio of the surface areas?
- What is the ratio of the volumes?



Determine the distance between each pair of points in space. Then determine the coordinates of the midpoint M of the segment joining the pair of points.

- the origin and $(0, -3, 5)$
- the origin and $(-1, 10, -5)$
- the origin and $(9, 5, -7)$
- $(-2, 2, 2)$ and $(-3, -5, -4)$
- $(9, 3, 4)$ and $(-9, -7, 6)$
- $(8, -6, 1)$ and $(-3, 5, 10)$

25. **STANDARDIZED TEST PRACTICE** A rectangular prism has a volume of 360 cubic feet. If the prism has a length of 15 feet and a height of 2 feet, what is the width?

(A) 30 ft

(B) 24 ft

(C) 12 ft

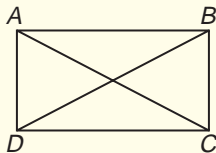
(D) 7.5 ft

SOL/EOC Practice

Part 1 Multiple Choice

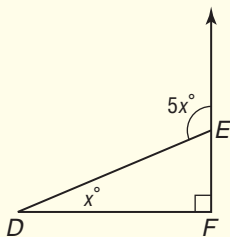
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. $ABCD$ is a rectangle. What is the relationship between $\angle ACD$ and $\angle ACB$? (Lesson 1-6)



- (A) They are complementary angles.
- (B) They are perpendicular angles.
- (C) They are supplementary angles.
- (D) They are corresponding angles.

2. What is the measure of $\angle DEF$? (Lesson 4-2)

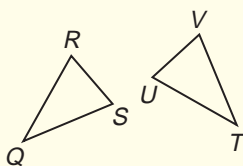


- (A) 22.5
- (B) 67.5
- (C) 112.5
- (D) 157.5

3. Two sides of a triangle measure 13 and 21 units. Which could be the measure of the third side? (Lesson 5-4)

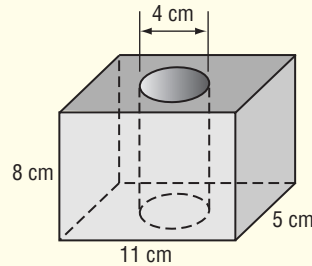
- (A) 5
- (B) 8
- (C) 21
- (D) 34

4. $\triangle QRS$ is similar to $\triangle TUV$. Which statement is true? (Lesson 6-3)



- (A) $m\angle Q = m\angle V$
- (B) $m\angle Q = m\angle S$
- (C) $m\angle Q = m\angle T$
- (D) $m\angle Q = m\angle U$

5. The wooden block shown must be able to slide onto a cylindrical rod. What is the volume of the block after the hole is drilled? Round to the nearest tenth. (Lesson 13-1)

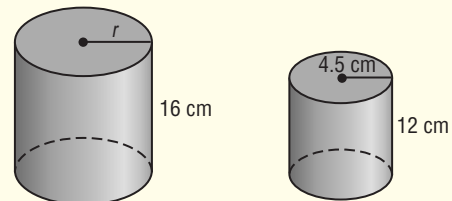


- (A) 100.5 cm^3
- (B) 339.5 cm^3
- (C) 402.0 cm^3
- (D) 440.0 cm^3

6. The circumference of a regulation soccer ball is 25 inches. What is the volume of the soccer ball to the nearest cubic inch? (Lesson 13-3)

- (A) 94 in^3
- (B) 264 in^3
- (C) 333 in^3
- (D) 8177 in^3

7. If the two cylinders are similar, what is the volume of the larger cylinder to the nearest tenth of a cubic centimeter? (Lesson 13-4)



- (A) 730.0 cm^3
- (B) 1017.4 cm^3
- (C) 1809.6 cm^3
- (D) 2122.6 cm^3

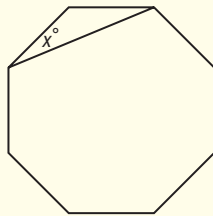
8. The center of a sphere has coordinates $(3, 1, 4)$. A point on the surface of the sphere has coordinates $(9, -2, -2)$. What is the measure of the radius of the sphere? (Lesson 13-5)

- (A) 7
- (B) $\sqrt{61}$
- (C) $\sqrt{73}$
- (D) 9

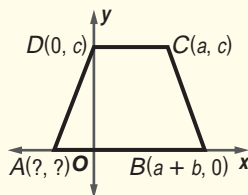
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

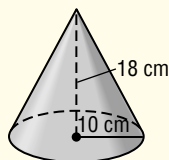
9. Find $\frac{12z^5 + 27z^2 - 6z}{3z}$. (Prerequisite Skill)
10. Sierra said, "If math is my favorite subject, then I like math." Carlos then said, "If I do not like math, then it is not my favorite subject." Carlos formed the ? of Sierra's statement. (Lesson 2-3)
11. Describe the information needed about two triangles to prove that they are congruent by the SSS Postulate. (Lesson 4-1)
12. The figure is a regular octagon. Find x . (Lesson 8-1)



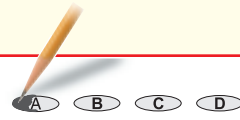
13. $ABCD$ is an isosceles trapezoid. What are the coordinates of A ? (Lesson 8-7)



14. What is the volume of the cone? (Lesson 13-2)



Test-Taking Tip



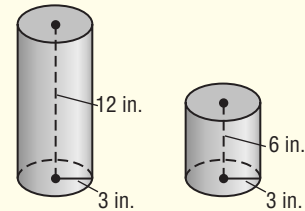
Question 6

Sometimes more than one step is required to find the answer. In this question, you need to use the circumference formula, $C = 2\pi r$, to find the length of the radius. Then you can use the surface area formula, $T = 4\pi r^2$.

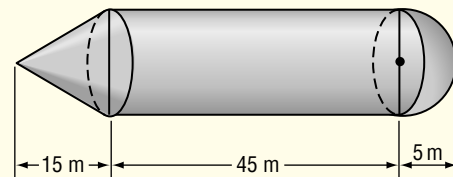
Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

15. A manufacturing company packages their product in the small cylindrical can shown in the diagram. During a promotion for the product, they doubled the height of the cans and sold them for the same price.



- a. Find the surface area of each can. Explain the effect of doubling the height on the amount of material used to produce the can. (Lesson 12-4)
- b. Find the volume of each can. Explain the effect doubling the height on the amount of product that can fit inside. (Lesson 13-1)
16. Engineering students designed an enlarged external fuel tank for a space shuttle as part of an assignment.



What is the volume of the entire fuel tank to the nearest cubic meter? Show your work.

(Lesson 13-1)



